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James Best | Keshav Dogra

FEDERAL RESERVE BANK *of* NEW YORK

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James Best and Keshav Dogra
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Abstract

Wealthier individuals have stronger incentives to seek higher returns. We investigate theoretically the effect this has on long-run wealth inequality. Incorporating capital management into a standard Ramsey-Cass-Koopmans model generates substantial long-run inequality: the majority of the population works and holds no capital, while a small minority holds a large amount of capital and manages it full-time. Counterintuitively, financial innovations or policies that reduce return differentials increase long-run wealth inequality. Egalitarian steady states may exist, but are inefficient and unstable: a small concentration in capital ownership causes a transition to an unequal steady state. Capital management introduces a novel equity-efficiency tradeoff: scale economies make it efficient for a few individuals to manage capital full-time, but under laissez-faire this generates substantial inequality. A utilitarian planner would instead instruct a few individuals to manage capital on behalf of society and transfer most of their income to the workers.

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Dogra: Federal Reserve Bank of New York (email: keshav.dogra@ny.frb.org). Best: Carnegie Mellon University (email: jabest@andrew.cmu.edu). The authors thank Sushant Acharya, Martin Ellison, Mike Elsby, Basile Grassi, Xiaowen Lei, Alistair Macaulay, John Moore, Ali Shourideh, Chris Sleet, Bill Zame, and Ariel Zetlin-Jones for comments and feedback.

1 Introduction

Wealth is extremely unequally distributed, across advanced economies. One cause of wealth inequality is that richer individuals earn higher average rates of return, even within asset classes (Fagereng et al., 2020). Understanding what causes these rate of return differentials is important for evaluating policies designed to address wealth inequality.

One explanation of these return differentials is that there is a cost, in terms of time or money, to managing one's wealth efficiently. Managing wealth can take various forms, from searching among different financial products, to starting and managing one's own business; all these activities increase the return on wealth, at a cost. Crucially, wealthier individuals have a stronger incentive to pay these costs to obtain higher returns (Arrow (1987)). Suppose that by spending all your time researching alternative investments, rather than investing passively, you can increase your annual return on wealth by 5 percentage points. If you have a billion dollars to invest, this increases your capital income by \$50 million, which may well justify managing your wealth full time. But if your wealth is \$10,000, your capital income only increases by \$500; you would earn more by working full time. Indeed, some have suggested that financial innovations such as low-fee index funds and robo-advisors could give low-wealth individuals (who cannot afford to manage their wealth full time) access to the high returns and financial advice previously restricted to the rich – narrowing the return differentials and reducing wealth inequality (Philippon, 2019; Laboure and Deffrennes, 2022; Reher and Sokolinski, 2023; D'Acunto and Rossi, 2023). In this paper, we explore theoretically the effect of capital management on long-run wealth inequality, and what this implies for the role of financial innovations and policy interventions in reducing inequality.

We incorporate capital management into an otherwise standard Ramsey-Cass-Koopmans model without shocks; households are identical except for their initial wealth. Households allocate their time between working for a wage and managing their capital. Spending e units of time managing your capital yields a return of $p(e) \times R$, where R is the marginal product of capital and p is an increasing function. (In the standard Ramsey model, we would have $p(e) = 1$ for all e.) Capital management represents the various ways in which individuals spend time developing better plans for the allocation of their wealth (e.g. searching among alternative financial products or managing a firm). Plans are nonrival – the same plan can simultaneously be used to manage one's first million dollars and one's second million – but excludable and nontradable. Thus, there are increasing returns to management and capital taken together *at the individual level*: if an individual doubles both her wealth, and the time spent managing her wealth, she more than doubles her income.

These increasing returns lead individuals with more capital to spend more time managing their capital, earning a higher effective return. This amplifies wealth inequality even in partial equilibrium. Rich individuals know that they will be rich in the future, will earn a higher return, and thus choose to accumulate wealth. Poor individuals know that they will be poor in the future, will not find it optimal to earn a high return on their capital, and thus choose to dissave. In general equilibrium, as rich individuals come to consume a larger share of these economy's capital stock, it is these individuals who determine the aggregate marginal product of capital, accumulating wealth until they reach a steady state. But if rich individuals are willing to keep their consumption constant in steady state, poorer individuals, who earn a lower rate of return, must have declining consumption, and must eventually reach zero wealth, becoming a propertyless worker.

These mechanisms create substantial wealth inequality in steady state. The majority of the population has zero capital (since low effective interest rates discourage accumulation) and works full time for a wage. All capital is held by a small minority, who manage their capital full time. The minimum gap between the poorest capitalist and a worker with zero capital is substantial. Somewhat counterintuitively, the lower the returns to capital management, the higher the minimum level of wealth inequality. To understand why, recall the example above. If managing one's capital full time only generates an increase of 50 basis points, rather than 5 percentage points, then an individual must be even richer to make managing capital full time worthwhile. In partial equilibrium, this might encourage full-time capital managers to dissave, reducing inequality. In general equilibrium, however, this capital decumulation would raise the marginal product of capital, encouraging accumulation up to a level where some individuals are rich enough to make full-time management worthwhile. Thus lower returns to capital management generate higher inequality. In fact, as the return to capital management goes to zero, wealth inequality explodes, since an individual must be richer and richer to make management worthwhile.[1]

This suggests that policies or financial innovations which reduce return differentials, such as financial literacy programs or robo advisors, may actually *increase* long-run wealth inequality. Similarly, the introduction of low-fee index funds led fees on equity mutual funds to fall from over 2% of assets in 1980 to around 1% in 2007 (Greenwood and Scharfstein, 2013). One might think that giving people with middling amounts of wealth access to higher returns would help reduce wealth inequality; yet, over the last four decades, these improvements in financial services have coincided with increasing US wealth inequality. In our model, more efficient financial intermediation has similar effects to a reduction in the returns to capital management: it *increases* long-run wealth inequality.[2]

How then can wealth inequality be curtailed in an economy where capital management is pos-

[1] Indeed, when the marginal return to effort for full-time managers *equals* zero (meaning that, in the cross section, a slightly richer individual always spends slightly more time managing capital) no steady state of this kind exists. Instead, in equilibrium, inequality is increasing forever. As interest rates sink to their long-run level, one by one, individual capital managers find that it is no longer worthwhile to manage full time, instead dropping out and becoming workers. Thus the measure of capital managers goes to zero, and the measure of workers goes to one. At the same time, the *average* wealth of the remaining capital managers goes to infinity, so that the aggregate capital stock converges to a constant. Capital management can generate ever-increasing wealth inequality, even absent shocks or other structural changes.

[2] How significant this mechanism is for explaining the increase in U.S. wealth inequality is an empirical question beyond the scope of this paper. Indeed, despite being able to access the same innovations, other advanced economies have not observed the same increase in wealth inequality seen in the U.S. We only wish to highlight the potentially perverse effects of improving the access of lower wealth households to financial opportunity.

sible? In economies with increasing returns, long-run outcomes can depend on initial conditions, so one might wonder whether a one-off redistribution of wealth can *permanently* reduce wealth inequality. The answer is mixed. If the return to zero effort, $p(0)$, is sufficiently high, there exist steady states in which no individual actively manages capital at all, leading to a relatively low aggregate capital-labor ratio. In this low-capital world, the marginal product of capital is high enough that individuals are willing to maintain their level of wealth even though they earn the low returns associated with unmanaged capital. Such steady states may be very egalitarian: indeed, it is possible for all individuals to have the same wealth. Thus, a sufficiently radical one-time levelling of wealth would indeed lead to egalitarian long-run outcomes.

However, such egalitarian steady states, even when they exist, are both unstable and inefficient. If we introduce an arbitrarily small measure of rich individuals into an egalitarian steady state, they disrupt the equilibrium and create a transition to one of the unequal steady states described above. In contrast, the unequal steady states are stable. If we add or subtract a small measure of wealthy individuals to an unequal steady state, we simply converge to another, essentially similar, unequal steady state. Thus, a one-time wealth redistribution is not a reliable way to reduce wealth inequality in the presence of shocks.[3]

Moreover, capital management introduces a novel tradeoff between equity and efficiency: increasing returns to scale in capital management make it more efficient for a few individuals to manage capital full-time, while the rest of society works, than for everyone to manage capital part time. Since the minority of capital managers earns a higher rate of return in equilibrium under laissez-faire, they will come to own all of society's capital stock in the long run, leading to substantial wealth inequality. But a steady state with equally distributed capital, while more equitable, is inefficient since small-scale capitalists manage their capital less well, while wastefully duplicating each other's managerial labor. Instead, a utilitarian planner would like a small minority to manage capital on behalf of the whole of society and transfer their capital income to the workers. This can be broadly interpreted as a social wealth fund which pays a dividend to all citizens (Meade, 1964; Atkinson, 2015; Bruenig, 2018), such as the Alaska Permanent Fund. Such an arrangement exploits scale economies arising from the nonrival nature of plans while sharing the benefits.

The rest of the paper is structured as follows. The remainder of this section discusses related literature. Section 2 presents the model. Section 3 discusses inequality and (utilitarian) welfare in inegalitarian steady states. Section 4 discusses egalitarian steady states. Section 5 introduces financial intermediation. Section 6 discusses optimal policy. Section 7 concludes.

[3]One implication of this instability is that a shock causing concentration of wealth in the hands of a few can have very large long-run effects on the utility of all agents. These differ in direction and magnitude depending on agents' initial (at the time of the shock) wealth levels. This presents a strong reason for people to care about regressive movements in the wealth distribution even when they are not affected directly by the initial redistribution.

1.1 Literature review

Arrow (1987) was, to our knowledge, the first to make the point that when individuals can increase their return on investments by acquiring information on rates of return, rich individuals will purchase more information and thus will enjoy a higher rate of return, since the value of information is greater for them. Consequently, the distribution of final wealth will be more unequal than the distribution of initial wealth. Peress (2004) extends this analysis, studying the effect of household wealth on the demand for risky assets in a static general equilibrium model. Arrow also discusses an alternative hypothesis which also predicts that wealth and rates of return will be positively correlated, namely that individuals with lower costs of information acquisition will enjoy higher returns and accumulate more wealth. Kacperczyk et al. (2018) quantitatively evaluate the second hypothesis within a general equilibrium model, and ask whether it can account for the increase in U.S. capital income inequality in recent decades. Our focus is instead on Arrow's first hypothesis – even if households have the same ability to acquire information, richer individuals have an *incentive* to buy more information – and on its dynamic general equilibrium implications.

Lei (2019) also considers Arrow's first hypothesis, in a quantitative partial equilibrium model where agents purchase signals about the return to risky assets, reducing uncertainty and facilitating higher investment in these higher return assets. She finds that a fall in the cost of information acquisition accounts for two-thirds of the rise in the top 1% wealth share. Similarly, Lusardi et al. (2017) studies a quantitative partial equilibrium model in which individuals can invest in a stock of financial knowledge, allowing them to receive a higher expected return from a 'sophisticated' investment technology. Macaulay (2021) studies the response to fiscal expansions in an economy where it is costly to process information about asset returns, leading rich households to earn higher returns. We view the $p(e)$ function in our model as a tractable way of capturing many such mechanisms through which time or money spent 'managing wealth' yields a higher return. This allows us to provide analytical results on the relationship between the return to management, the efficiency of the financial sector, and long-run wealth inequality in general equilibrium. General equilibrium effects amplify the importance of Arrow's first hypothesis; they also matter for the effect of policies or structural changes. In particular, while narrowing returns differentials may reduce inequality in partial equilibrium, it increases inequality in general equilibrium.

Campanale (2007) studies a quantitative general equilibrium model in which the rate of return on savings is increasing in a household's wealth. He finds that this generates a substantial and empirically plausible increase in wealth inequality relative to a standard model. Benhabib et al. (2019) find that allowing rates of return on wealth to be increasing in wealth helps account for the skewed wealth distribution in the United States. One difference relative to these studies is that in our model, the positive relation between individual wealth and the return on savings is a result, rather than a technological assumption. As such, the nature of this relation depends on factor prices in general equilibrium. In this regard, our analysis is more similar to that of McKay (2013), who considers a general equilibrium model in which households can exert effort to gain a higher return on savings. Unlike all these analyses, that since we study the general equilibrium effect of increasing returns analytically, rather than quantifying it numerically, we can characterize the sensitivity of long-run inequality to the technology for capital management.

Two recent papers document that wealthier individuals earn higher returns empirically. Fagereng et al. (2020) find using Norwegian admininistrative data that richer households earn higher returns even within each asset class, possibly indicating that they spend time or money acquiring superior information about which assets to invest in, consistent with our model. Using Swedish data, Bach et al. (2020) also document systematic return differentials between richer and poorer households, but find that return differentials largely reflect compensation for higher risk-taking. While this is inconsistent with the hypothesis that rich households acquire information about the return on particular stocks, it would be consistent with a looser interpretation of our model in which individuals begin with a pessimisic prior about the benefits of investing in the stock market as an asset class, and must spend resources to acquire more accurate information. Indeed, also using Swedish data, Briggs et al. (2021) find that pessimistic beliefs regarding equity returns are the most likely explanation of limited stockmarket participation. Motivated by this empirical evidence, we study the effects of increasing returns to wealth in a general equilibrium setting.

Finally, on the technical side, we draw on the literature on optimal growth with non-convex production functions (Dechert and Nishimura (1983)): the individual decision problem for a household in our model is isomorphic to the problem faced by a social planner in the literature on non-convex optimal growth – even though our aggregate production function features decreasing returns.

2 Model

Time is discrete and there is no uncertainty. There is a continuum of households with measure 1 indexed by $i \in [0,1]$. Households have preferences

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

Each household has a unit endowment of time, which they can allocate between managing capital, and working in the labor market. At each date t, a household with capital $k_t^i > 0$ operates a firm with a constant returns to scale production technology $F(K,L)$.

Assumption 1. *F is twice continuously differentiable and homogeneous of degree 1. Both factors are necessary: $F(K,0) = F(0,L) = 0$ for any K, L. $F_L > 0$, $F_K > 0$, $F_{KK} < 0$, $F_{LL} < 0$. The Inada conditions hold: $\lim_{\theta \to 0} F_K(\theta,1) = \infty$, $\lim_{\theta\to\infty} F_K(\theta,1) = 0$, $\lim_{\theta\to 0} F_L(\theta,1) = 0$, $\lim_{\theta\to\infty} F_L(\theta,1) = \infty$ where $\theta := \dfrac{K}{L}$ denotes the effective capital-labor ratio.*

Households are identical except for their initial wealth k_0^i; without loss of generality, we order them so that k_0^i is nondecreasing in i. We deliberately abstract from the typical sources of heterogeneity studied in the wealth inequality literature in order to facilitate a complete analytical characterization of our environment with increasing returns to individual wealth. Even without exogenous heterogeneity, our model generates substantial wealth inequality; we discuss below how adding these features would affect our results.

A household with $k_t^i > 0$ also chooses how much time $e_t \in [0,1]$ to spend managing the firm, supplying her remaining $1 - e_t$ units of time as a wage laborer. Households with $k_t^i = 0$ work full time as wage laborers. Spending more time managing capital increases the productivity of capital; k units of 'raw' capital, managed with intensity e, generate $p(e)k$ units of effective capital. Formally:

Assumption 2. *$p : [0,1] \to \mathbb{R}_+$ satisfies $p(0) \in (0,1)$, $p(1) = 1$, $p'(e) > 0$ for $e < 1$, $p''(e) < 0$.*

Labor demand There is a market in labor, but no market in capital. Given the level of managerial effort, the capital stock, and the wage w, the firm hires labor ℓ to maximize profits:

$$\pi(e,k) = \max_{\ell} F(p(e)k, \ell) - w\ell$$

Note that the same individual may both hire labor as a firm and supply labor as a wage laborer; in this case, it is immaterial whether we assume the individual works for her own firm or another firm. Optimal hiring yields the first order condition $F_L(p(e)k, \ell) = w$. In equilibrium wages are equal across firms, equalizing effective capital labor ratios. Since F_L is homogenous of degree zero, multiplying both sides by the aggregate effective labor stock $L_t = \int \ell_i di = \int (1 - e_i) di$ and dividing by ℓ_i yields $F_L(p(e_i)k_i, \ell_i) = F_L\left(p(e_i)\frac{k_i}{\ell_i}L, L\right) = F_L(K, L)$, where we define the effective capital stock $K_t = \int p(e_t^i)k_t^i di$. Similarly, the marginal product of effective capital is equalized across firms:

$$R = F_K(p(e_i)k_i, \ell_i) = F_K(K, L)$$

Lemma 1. *For optimal choice of $\ell^* = \ell(e,k)$, the firm's profit is*

$$\pi(e,k) := \max_{\ell} F(p(e)k, \ell) - w\ell = p(e)Rk$$

Proof. This follows from Euler's Theorem, since factor prices equal marginal products. □

'Capital management' e represents the many ways in which individuals with physical or financial resources at their disposal can spend time or effort to earn a higher return from these resources. Managers and entrepreneurs spend their time formulating and considering alternative corporate decisions and strategic directions for their organization, reviewing investment proposals, and evaluating senior employees. These processes inform their *plans* – decisions they make on how to deploy their resources – which are then implemented by employees. Similarly, high-net worth individuals (HNWIs) spend time evaluating alternative investment opportunities; this informs the portfolio allocation they ultimately choose. Since entrepreneurs and HNWIs choose to spend time managing their wealth, doing so must lead to a higher rate of return on wealth, at least in expectation. Our model is general enough that 'capital' can be interpreted as either physical capital or financial wealth, even though these are not the same thing.

A plan for the deployment of wealth is an *idea*, not an object (Romer, 1993). The standard replication argument tells us that if we double the *rival* resources deployed according to a given plan – the machines and labor used to produce a particular product, or the dollars invested in particular financial assets – we produce twice as much output or revenue. The same plan can simultaneously be used to manage the first million dollars and the second million. If instead we double the rival resources *and* spend twice as much time planning, producing a more profitable plan, we produce more than twice as much. As in Romer (1990), the nonrivalrous nature of ideas implies increasing returns to objects and ideas taken together. Crucially though, in our economy plans are not only excludable (they cannot be freely copied), but altogether nontradable (they cannot be sold or licensed). Thus, there are increasing returns *at the individual level*: an individual internalizes that if she doubles *her* wealth, and *her* time spent managing *her* wealth, she more than doubles her income.

Since plans are nonrival, it might be technically feasible (indeed, efficient) for 'lazy' entrepreneurs or HNWIs, with their own resources to deploy, to simply copy the plans of 'industrious' wealth managers who spend time researching their plans – allowing the lazy managers to do something else with their time. Or, if plans are excludable, industrious managers might be able to license their plans to lazy managers for a fee. But copying and licensing of plans only occurs to a limited extent. CEOs do not typically sell copies of their entire business plan to competing managers who then implement the same plan. Traders are often hesitant to share their trading strategies. These limits arise for various reasons. A plan developed by one manager (e.g. a business idea developed for a particular market) may not be useful to another manager. Managers may not be able or willing to license their plan for a fee, e.g. because they cannot prevent the licensee from reselling it or giving it away for free.[4] Plans may be experience goods rather than inspection goods: it is hard for a 'lazy' HNWI to check whether the portfolio plan sold by an 'industrious' HNWI is the product of careful or shoddy research. For simplicity, our model rules out spillovers, copying or licensing altogether; even if they occur to some extent, what matters is that a manager still earns a higher return by managing their own capital than by relying on others' research.

The $p(e)$ function is a tractable way of capturing the many ways in which richer individuals can spend time or effort to earn a higher rate of return on their wealth. There are many possible microfoundations of this function:

- *Managerial effort*: As in the literature on CEO incentives (e.g. Baker and Hall (2004); Edmans et al. (2009)) e can be interpreted directly as the 'effort' or cost of actions taken by managers or entrepreneurs organizing their business and increasing the value or productivity of their

[4]As we will see, a manager will strictly prefer that other managers do not use her plan for free, since free-riding erodes the return that she earns from the plan.

firm. Since the effect of actions such as a corporate reorganization or change in strategy scales with the size of the firm (i.e. the amount of capital k under management), the marginal product of effort $p'(e)Rk$ is increasing in firm size.

- *Portfolio choice*: as we show formally in Appendix F, e can be interpreted as the time and effort spent by households managing their portfolio, searching for investments with a higher return (McKay, 2013).

- *Costly information acquisition*: Suppose a manager must allocate capital among projects which transform 'raw capital' into 'usable capital' at a rate of 1 (for 'good' projects) or zero ('bad' projects). The manager receives a binary signal ('high' or 'low') such that the probability that the project is good, conditional on a high signal, is $p(e) \geq 1/2$; optimally the manager will allocate all capital to projects with a 'high' signal, implying that usable capital equals $p(e)k$. As in the literature on rational inattention (Sims, 2003; Van Nieuwerburgh and Veldkamp, 2009; Mondria, 2010; Kacperczyk et al., 2018), the manager can pay a cost e (here specified in terms of time) to acquire a more precise signal, i.e. $p'(e) > 0$ and $p(1) = 1$.

- *Financial intermediation*: As we show in Section 5, the choice between high and low values of e can be interpreted as the choice between managing capital yourself (and earning a high return at the cost of spending time managing) and lending to a financial intermediary (which costs less time, but earns a lower return due to various frictions). More generally, there exists a continuum of investment products (from cash and insured deposits, through passively managed index funds, to individually held stocks, private equity, and private businesses) which earn an increasingly higher average return, but require increasingly higher time, attention and effort. The key implication of our model, under this interpretation, is that richer households allocate a greater share of their wealth to asset classes which require more attention and yield higher returns.

The endogenous growth literature typically assumes research contributes to a stock of ideas which can be used to produce both consumption goods and ideas at all future dates, consistent with that literature's focus on ideas such as scientific discoveries, process innovations, and new product designs, which are important for long-term growth. Instead, we assume time spent managing capital at date t produces ideas that contribute to production at date t, but not at future dates (plans fully depreciate after one period). Many plans for the management of capital are ephemeral, contributing to production over a limited period of time. Business plans govern a particular year's activities; market research studies recent trends; hedge fund strategies often exploit short-term anomalies or opportunities that soon vanish. Of course, these distinctions are not absolute; the research behind one year's business plan may also inform planning and production in future years. Abstracting from these dynamic linkages by assuming full depreciation of plans simplifies the analysis considerably and allows us to focus on inequality rather than growth. Similarly, we abstract from direct externalities through which one manager's effort choice affects the productivity of other managers' capital. Plausibly, these could be either positive (spending time managing one's business generates ideas which others can learn from) or negative (developing better market-timing investment strategies reduces returns for other traders).

Finally, the literature on entrepreneurial effort and firm size, starting with Lucas (1978), assumes a degree of diminishing returns to scale, or "span of control". By assuming that $p(e)$ is linear in k, we focus on the less studied case of increasing returns at the individual level. Such linearity is quite natural if one interprets k as financial wealth: at least within some range, investors can purchase any volume of assets without appreciably impacting price. Nonetheless, one might reasonably assume there is some scale, k^{span}, above which investors and/or managers face decreasing returns. Formally, $p(e,k)$ depends on k as well as e, is constant in k for $k \leq k^{span}$, and decreasing for $k > k^{span}$. Provided that k^{span} is sufficiently large (above $\underline{k}$), our results would be largely unchanged in such a case, except that wealth inequality would be bounded above since no individuals would have $k > k^{span}$ in steady state. As one of the purposes of this paper is to study increasing returns, we have little interest in the case where k^{span} is small.

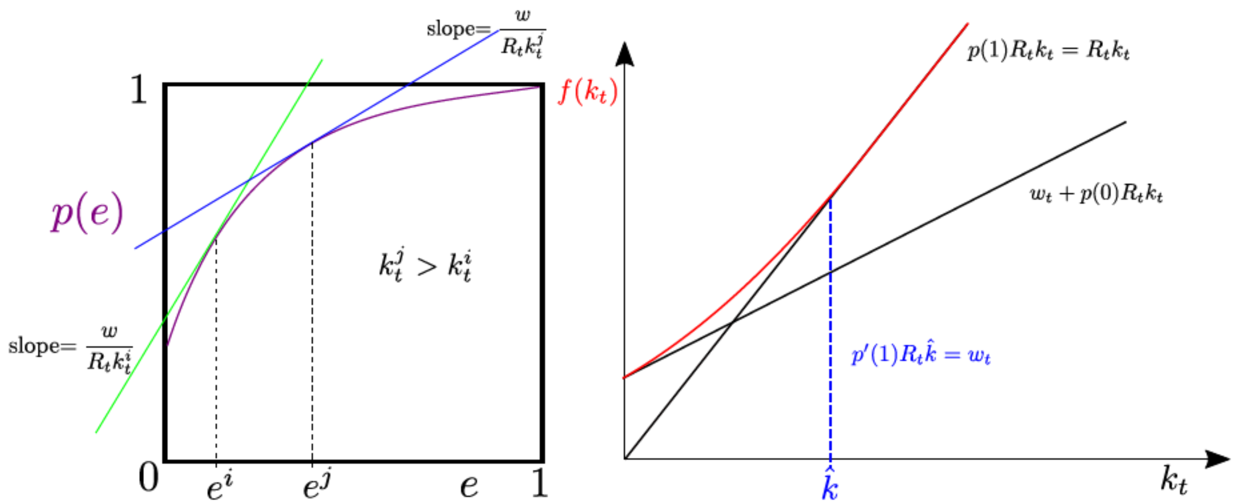
Capital management and labor supply Given a level of capital k_t^i, an individual must decide how much time to allocate to management versus working for a wage. They will choose e_t to solve the static decision problem

$$f_t(k_t) = \max_{e \in [0,1]} p(e)R_t k_t + (1-e)w_t$$

immediately implying that $f_t(k_t)$ is convex. When $p'' < 0$ and there is an interior solution, this implies the first-order condition $p'(e_t)k_t R_t = w_t$, which we can invert to yield $e_t(k_t) = p'^{-1}\left(\frac{w_t}{R_t k_t}\right)$. $e_t(k_t)$ is weakly decreasing in w_t and increasing in k_t and R_t. When the return to wage labor is relatively low, or the return to capital management is high, individuals optimally spend more time managing capital. Importantly, the individual's return to management is increasing in their wealth, i.e. their holdings of raw capital. An extra hour spent managing capital increases the *rate of return* on their total wealth, and thus has a larger effect on overall income for an individual with higher wealth. An hour managing a fortune yields a far greater reward than an hour managing a pittance.

This is illustrated in Figure 1a. The straight lines show an individual's 'preferences' over time spent managing capital, e, and 'capital effectiveness', $p(e)$. Higher e is disliked because it subtracts from labor income, and has cost w, while higher effectiveness is valued because it increases capital income, and has benefit $R_t^i k_t$. Thus income is increasing as we move northwest. The curved purple line shows the individual's constraint set, given by the function p which represents the technology for turning time into capital effectiveness. The steep green line illustrates the preferences of an individual i with relatively low wealth k_t^i; he finds it optimal to spend only a small amount of time managing capital. As we turn to a wealthier individual j with $k_t^j > k_t^i$, whose preferences are indicated by the flatter blue line, we increase the relative return to capital management, and so j

optimally chooses to spend more time managing her capital than i: $e_t^j > e_t^i$.



(a) Capital management and labor supply

(b) Increasing returns to raw capital

Figure 1. Capital management and labor supply

Given optimal capital management, income will be a convex function of raw capital $f_t(k_t)$, and the marginal return to raw capital is (weakly) increasing. This is illustrated in figure 1b, which plot income against raw capital. The flatter black line shows income as a function of wealth conditional on working full time and spending no time managing capital ($e = 0$); the steeper black line shows the same relationship when an individual manages capital full time and spends no time working ($e = 1$). Since both strategies are feasible, income under optimal capital management must lie above both lines, and is in fact the upper envelope of all such lines (for other values of $e \in (0, 1)$, not shown here), represented here by the red curve $f_t(k_t)$.

Note that when $k_t = 0$, it is clearly optimal to spend no time managing capital, so the red and (flatter) black lines coincide. More importantly, if wealth is sufficiently high it is optimal to manage capital full time, and never work for a wage. This will be true whenever wealth k_t is weakly greater than the *full-time capitalist* level of wealth $\hat{k}(w, R)$, defined by[5]

Definition 1. $\hat{k}(w, R) = \dfrac{w}{R_t p'(1)}$

In a dynamic context, as we will see, increasing returns will imply that the wealthy save more than the poor. If you will not be very rich tomorrow in any case, you will not manage capital very effectively in the future, and anticipating this low return, it may not be worthwhile to save today. In contrast, if you will be wealthy in the future, you will manage your portfolio very effectively; anticipating this high return, it may be worth saving more today. To demonstrate this, we turn to the individual's dynamic optimization problem.

Consumption and wealth accumulation Given an initial level of capital k_0 and a sequence of wages $\{w_t\}_{t=0}^{\infty}$ and returns on capital $\{R_t\}_{t=0}^{\infty}$, an individual chooses sequences of consumption,

[5] If $p'(1) = 0$, no such level $\hat{k}$ exists. We analyze both the cases $p'(1) > 0$ and $p'(1) = 0$ below.

capital and effort $\{c_t, k_{t+1}, e_t\}_{t=0}^{\infty}$ to solve

$$\max_{\{c_t,k_{t+1},e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t$$
$$\text{s.t. } k_{t+1} + c_t \leq R_t p(e_t) k_t + (1 - e_t) w_t$$
$$e_t \in [0,1],\ k_t \geq 0, \forall t,\ k_0 > 0$$

For simplicity we assume full depreciation. Note that if $p(0) = p(e) = 1$, this would simply be the problem of a household in a standard Ramsey-Cass-Koopmans model.

Given optimal time allocation, the first two constraints in the household problem can be summarized as

$$k_{t+1} + c_t \leq f_t(k_t)$$

As we just saw, $f_t(k)$ is a convex function. In steady state, $f_t(k) = f(k)$ and the problem of an *individual* household is isomorphic to the *social planner's problem* in the literature on optimal growth with nonconvex technology (Dechert and Nishimura (1983)), even though our aggregate production function features diminishing returns. These nonconvexities make the individual's problem nontrivial to solve, since first order conditions are not sufficient for optimality.

Equilibrium is defined as follows:

Definition 2. *An equilibrium is a sequence of prices and aggregate variables* $\{R_t, w_t, K_t, L_t\}_{t=0}^{\infty}$ *and consumption, effort and capital distributions* $\{c_t^i, e_t^i, k_{t+1}^i\}_{t=0}^{\infty}, i \in [0,1]$ *such that*

1. *for each* $i \in [0,1]$, $\{c_t^i, k_{t+1}^i, e_t^i\}_{t=0}^{\infty}$ *solves the individual's problem, given initial condition* k_0^i *and prices* $\{R_t, w_t\}_{t=0}^{\infty}$

2. *aggregate variables satisfy*

$$w_t = F_L(K_t, L_t)$$
$$R_t = F_K(K_t, L_t)$$
$$K_t = \int p(e_t^i) k_t^i di$$
$$L_t = \int (1 - e_t^i) di$$

Properties of equilibrium Since the individual's problem is nonconvex, in general it may have multiple solutions. The following Lemma narrows down the scope for multiplicity: multiple solutions can only exist at date zero, or after the individual hits zero wealth.

Lemma 2. *Take an optimal plan* $\{k_t\}_{t=0}^{\infty}$. *If* $k_1 > 0$, *the optimal choice of* k_2 *is unique starting from* k_1. *That is, no other plan* $\{k_0, k_1, k_2', k_3', ...\}$ *with* $k_2' \neq k_2$ *is also optimal.*

Proof. Suppose by contradiction $k_1 > 0$ and $\{k_0, k_1, k_2, ...\}$ and $\{k_0, k_1, k_2', ...\}$ were both optimal. From the Euler equations,

$$u'(f_0(k_0) - k_1) = \beta f_1'(k_1)u'(f_1(k_1) - k_2)$$
$$u'(f_0(k_0) - k_1) = \beta f_1'(k_1)u'(f_1(k_1) - k_2')$$

Since u' is strictly decreasing, this implies $k_2 = k_2'$. □

A similar result is the following:

Corollary 1. *If $k_1 > 0$, there is at most one $k_0 \in \mathbb{R}_+$ such that k_1 is optimal given k_0.*

Proof. Suppose that $k_1 > 0$ is optimal given both k_0 $\tilde{k}_0$. We know from Lemma 2 that there is a unique k_2 which is optimal given k_1 and a correspondingly unique value for consumption, $c_1 = f_1(k_1) - k_2$. So $u'(f_0(k_0) - k_1) = \beta f_1'(k_1)u'(c_1) = u'(f_1(\tilde{k}_0) - k_1)$, which implies $k_0 = \tilde{k}_0$. □

An important property of solutions to the individual problem is that richer individuals save more, in the following sense.

Lemma 3 (One Period Sorting). *If $\{k_t^l\}$, $\{k_t^h\}$ are optimal given some k_0^h, k_0^l and if $k_t^h > k_t^l$, then $k_{t+1}^h \geq k_{t+1}^l$, with strict inequality unless $k_{t+1}^h = k_{t+1}^l = 0$.*

Proof. See Appendix A. □

Steady states are equilibria in which aggregate and individual variables, for each individual $i \in [0, 1]$, are constant over time:[6]

Definition 3. *A steady state is a collection of prices and aggregate variables $\{R, w, K, L\}$ and consumption, effort and capital policies $\{c^i, e^i, k^i\}$ such that the constant sequences $R_t = R, w_t = w$, etc. for all $t = 0, 1, ...$ constitute an equilibrium.*

Applied to steady states, Lemma 3 implies that individual trajectories are monotonic.

Corollary 2 (Monotonic trajectories). *If $R_t = R, w_t = w$ for all t, optimal paths $\{k_t\}$ are monotonic. That is, an optimal path is either increasing forever, constant, decreasing forever, or decreasing and then zero forever.*

Proof. This follows immediately from Lemma 3. □

[6] In our deterministic environment, it is straightforward to show that this definition is equivalent to requiring that the distribution of wealth is constant over time, without imposing that each individual maintains constant wealth.

3 Steady state inequality and welfare

Next we characterize steady states of this economy in which prices and aggregate variables are constant over time. In this section we show how the mechanisms described above generate substantial inequality in steady state. Throughout this section we make the following assumption.

Assumption 3. $p'(1) > 0$

As shown above, this implies that there exists a level of wealth $\hat{k}(w_t, R_t)$ above which all individuals manage their capital full time. Equivalently, increasing returns to wealth only obtain up to some level; after that, there are constant returns to wealth. This level of wealth is endogenous (it depends on factor prices) and can be arbitrarily high as $p'(1)$ becomes small. In Section 3.3 we analyze the case $p'(1) = 0$.

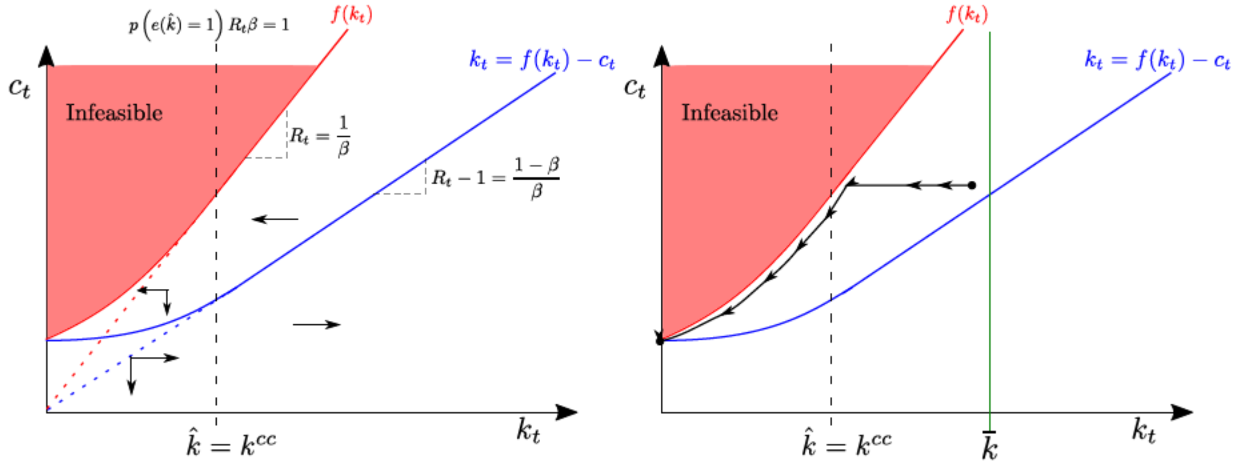
We will shortly consider steady states in which there exist some full-time capitalists setting $e^i = 1$. In order for these capitalists to maintain constant consumption, we must have $\beta R = 1$. To see this, note that with constant factor prices, necessary conditions for individual optimality are

$$\frac{c_{t+1}}{c_t} \geq \beta f'(k_{t+1}), k_{t+1} \geq 0, \text{ at least one strict equality}$$
$$k_{t+1} = f(k_t) - c_t$$

In steady state, each individual's behavior is characterized by a fixed point of this system. Thus for individuals with $k^i > 0$ who manage capital full time, we must have $\beta f'(k^i) = \beta R = 1$. This further implies there can be no part-time capitalists in steady state: if $e^i < 1$, $\beta R p(e^i) < 1$, and such individuals would have declining consumption, contradicting the definition of steady state. Intuitively, when full-time capitalists accumulate wealth until they are content to maintain constant consumption, they drive down the rate of return so much that it is unprofitable to manage capital in the long run unless one does so full-time.

These optimality conditions describe a dynamic system, which we can display in a phase diagram, Figure 2a. Again, since the individual problem is nonconvex, these conditions are necessary but not sufficient for optimality. The vertical dashed line shows the full-time capitalist level of wealth $\hat{k}(w, R)$. To the right of this line, individuals maintain constant consumption. To the left of this line, consumption must be declining along an optimal plan. The blue line shows the level of consumption corresponding to rolling over one's current level of wealth, $c = f(k) - k$. If consumption is above this level, wealth is declining, indicated by the arrows pointing left; if consumption is below this level, wealth is increasing (arrows pointing right). The red curve shows the maximum feasible level of consumption.

From this diagram, it is clear that if an individual chooses $k_1 < \hat{k}$, to the left of the dashed line, two things can happen: either the individual reaches zero wealth and consumes the wage thereafter, or they enter the bottom left quadrant, begin accumulating capital, and eventually reach the bottom



(a) Trajectories satisfying optimality conditions

(b) Optimal decumulation of capital

Figure 2. Phase diagram, $\beta R = 1$

right quadrant, increasing capital forever and maintaining constant consumption. Clearly the latter course of action is not optimal, since at any point, the individual could jump up to the blue line and stay there, enjoying higher consumption in every period. So it must be that if $k_1 < \hat{k}$, the individual converges to zero wealth.

If $k_1 > \hat{k}$, again there are two possibilities. We can rule out the possibility that individuals choose consumption below the blue line as above. They may choose consumption on the blue line, maintaining constant consumption $c = (1-\beta)Rk$ forever. Or they may choose consumption above the blue line, decumulating wealth and eventually entering the lower right quadrant, decumulating wealth, and becoming a full-time worker. This second possibility is depicted by the black line in Figure 2b. In fact, it will turn out that such a path is indeed optimal when capital is not too high, below some level $\underline{k}$.

Such a path cannot be optimal, however, when initial wealth is too high. Graphically, it is clear that starting from a very high level of wealth, a trajectory which features constant consumption at a level above $f(k_0) - k_0$ will eventually crash into the infeasible region indicated by the red shaded area – i.e. such a plan must feature a steep drop in consumption in order to prevent consumption from exceeding income. Given households' preference for consumption smoothing, such a large drop in consumption is – eventually – undesirable.

To sum up, we have seen that in any steady state with $\beta R = 1$, the low return on capital induces individuals with an intermediate level of wealth to decumulate, eventually ceasing to manage their capital at all and become full-time workers. Proposition 1 formalizes this statement and shows that such steady states exist.

Proposition 1. *Under assumption 3, there exist steady states in which $\beta R = 1 = \beta F_K(\theta, 1) = 1$, $w = F_L(\theta, 1)$, where $\theta = K/L$ denotes the effective capital-labor ratio.*

There exists $\underline{k}$ such that in all such steady states, the support of the steady state distribution is a subset of $\{0\} \cup [\underline{k}, \infty)$ with positive mass at both zero and $[\underline{k}, \infty)$. Furthermore, any distribution *with support in* $\{0\} \cup [\underline{k}, \infty)$ *is a steady state of the kind described, provided that* $K/L = \theta$ *where* K *denotes the average level of capital* $\int k^i di$ *and* L *denotes the mass of individuals with zero capital.*

We have $\underline{k} \in [\hat{k}(w, R), \tilde{k}]$ *where the bounds are defined by*

$$\tilde{k} = \max\left\{\frac{1}{p(0)}, p'(1)(1-\beta)^{-1/\beta}\right\}\frac{\hat{k}}{1-\beta}$$

and $\hat{k}$ *is described in Definition 1.*

Individuals with positive capital manage capital full time $(e = 1)$ *and consume* $(1-\beta)Rk$*. Individuals with zero capital work full time* $(e = 0)$ *and consume their wage,* $c = w$*. The measure of workers* L *satisfies*

$$L \geq \frac{\hat{k}}{\hat{k} + \theta}.$$

Proof. See Appendix B. □

In steady state, individuals segregate into two classes: full time capitalists who never work, and full time workers with no capital. We have a lower bound on the wealth of the full time capitalists.

Recall that a standard Ramsey-Cass-Koopmans model with $p(0) = p(e) = 1$ would permit *any* distribution of wealth to be an equilibrium, provided that the aggregate capital stock satisfied $\beta F_K(K, 1) = 1$. In contrast, Proposition 1 rules out some of these steady states, namely those in which some individuals have wealth in the interval $(0, \underline{k})$. Steady states must feature a 'hollowed out middle class'. Some permissible steady states may be more equal, some less equal, but all feature a certain minimum amount of inequality.

In particular, notice that while the effective capital-labor ratio θ is pinned down, the quantity of labor L (which, given that all workers work full-time, is just the number of wage laborers) not uniquely determined. The fraction of full time workers *is* bounded below: in any steady state of the kind described, the working class must constitute a sizeable majority of the population.[7] However, above this lower bound, there exist steady states with a relatively smaller working class, a relatively large capitalist class with smaller capitalists, and lower aggregate output. There also exist steady states in which the working class constitutes almost the entire population, the capitalist class is small in number but very wealthy in a per capita sense, and output is relatively high. A full welfare ranking of these steady states is deferred to subsection 3.2.

We have deliberately abstracted from the standard ingredients used in the wealth inequality literature – ex ante and ex post skill heterogeneity, finite lives, and so forth – to permit a complete analytical characterization of steady state outcomes. Incorporating these features would not change our main result that capital management generates substantial wealth inequality, but would soften some of the model's extreme predictions, at the cost of reducing analytical tractability. Suppose for example that individuals face shocks to their productivity as workers, following Aiyagari (1994),

[7] More precisely, the fraction of full-time workers must be at least as large as the labor share of income. Assuming the labor share exceeds 0.5, this that implies the working class constitutes a majority.

and suppose for simplicity that $p(e)$ is linear, so everyone either works or manages capital full time. This would modify the model's extreme prediction that workers hold no wealth: instead, workers will accumulate some capital for precautionary purposes, even though they earn a lower rate of return $p(0)R$ than capitalists, who earn R. But, as in Aiyagari (1994), there will exist a maximum level of precautionary wealth which workers choose to hold. Provided that this level is not too high, relative to $\underline{k}$, there may still exist some much richer capitalists who never work and manage their capital full time. Thus, the wealth distribution would still feature a 'hollowed out middle class', with full-time managers accumulating the lion's share of wealth and earning higher returns.[8]

3.1 The effect of return differentials on wealth inequality

We have shown that the possibility of capital management gives rise to inequality in steady state: there is a gap between the wealth of the poorest capitalist and the wealth of any worker. Next we ask how the size of this wealth gap depends on the return to capital management $p(e)$, or in other words the return differential $\frac{R_t p(1)}{R_t p(e)} = \frac{1}{p(e)}$ between a full-time manager and an individual spending only $e < 1$ of their time managing capital. The following Lemma describes the effect of an upward shift in the $p(e)$ function; given our normalization $p(1) = 1$, this reduces the return to capital management, and narrows the rate of return differential between full- and part-time managers.

Lemma 4. *Consider two economies with different returns to effort, p^0 and p^1, with $p^0(1) = p^1(1) = 1$, $p^1(e) \geq p^0(e)$ for all $e \in [0, 1]$. Then the corresponding minimum individual capital level is higher under p^1: $\underline{k}^1 \geq \underline{k}^0$.*

Proof. By definition, $\underline{k}$ is the lowest level of capital such that a 'Mertonian strategy' with $e = 1$ and $c = (1 - \beta)Rk$ is optimal. Increasing p from p^0 to p^1 leaves the return to a Mertonian strategy unchanged, while increasing the return to any other strategy. The result is immediate. □

A reduction in the return to capital management increases the minimum individual capital level $\underline{k}$, and thus increases the minimum number of propertyless workers in steady state, $1 - \underline{L} = \frac{\theta}{\hat{k}+\theta}$. A narrower rate-of-return gap makes it possible to manage capital more efficiently without doing so full time. But in the long run, the only capital managers are full-time managers; given that full-time managers have driven down their return to β^{-1}, the return to part-time management is never high enough to make this a worthwhile long-term option for the individual. However, this change *does* make it more attractive to manage capital part-time for a short time. Recall that from Figure 2b, some wealthy individuals found it optimal to decumulate wealth, eventually falling below $\hat{k}$ and managing part-time, before ultimately becoming propertyless wage laborers. Since this deviation involves a spell of part-time management, an increase in the productivity of part-time management makes the deviation more attractive, so that some individuals – who would otherwise have remained capitalists forever – will now decide to decumulate.

[8] Similarly, while the assumption of log utility considerably simplifies some of the proofs, assuming more general concave utility would not qualitatively change our results.

In general, the set of possible steady states depends on the whole function $p(e)$. However, it turns out that the long-run properties of this economy are very sensitive to one particular aspect of this function, namely the marginal return to effort for full time managers, $p'(1)$.

Proposition 2. *Consider a sequence of functions $\{p_n\}$ with $p'_n(1) \to 0$. As $n \to \infty$, $\hat{k} \to \infty$, the wealth gap between the worker and the poorest capitalist goes to infinity, and $L \to 1$: the measure of capitalists goes to zero, and the average wealth of each capitalist goes to infinity.*

Proof. Since $\hat{k} = \dfrac{w}{Rp'(1)}$ and w, R are independent of p (given that $p(1) = 1$), it follows that as $p'(1) \to 0$, $L \to 1$. The minimum possible level of individual steady state capital, $\underline{k}$, is always greater than $\hat{k}$, and so it must diverge to infinity as well. The result follows. □

There is a simple static intuition for this result. If the marginal return to managing capital falls towards zero, individuals require an ever larger capital stock in order to dissuade them from managing wealth part time. If they have insufficient wealth, and choose to manage it part time, their consumption will decline over time and they will eventually become a propertyless worker. One can see in Figure 2b that as $\hat{k} \to \infty$, $\underline{k} \geq \hat{k}$ must do the same. This might seem to permit steady states in which the same number of capitalists each manage a higher and higher stock of capital. However, this increase in the capital stock would push the return on capital below β^{-1}, weeding out the smaller capitalists and resulting in a wealthier, but less numerous, capitalist class.

Various financial innovations and policy interventions can be broadly interpreted as reductions in the return to capital management which reduce differentials. More financially literate individuals earn higher rates of returns (Clark et al., 2017); financial literacy programs can be interpreted as interventions which raise the return earned by an individual who cannot afford to manage their wealth full-time. Some have argued that 'robo advisors' could provide wealth management services (typically available to wealthy clients) to less wealthy individuals, narrowing return differentials (Philippon, 2019; Reher and Sokolinski, 2023; D'Acunto and Rossi, 2023). More generally, while an explicit treatment of financial intermediation is deferred to Section 5, an increase in $p(e)$ is loosely analogous to financial innovations which make it easier for intermediaries to manage other people's money. This allows individuals of middling wealth to work full-time or part-time while earning, if not the same return as an actively managed portfolio, a higher return than was previously possible without becoming a full time manager.

It is often argued that, since these polices and interventions reduce return differentials, they must also reduce wealth inequality. Our result suggests a note of caution. In general equilibrium, narrowing return differentials might actually increase inequality in the long run, while decreasing the number of wealth-owners.

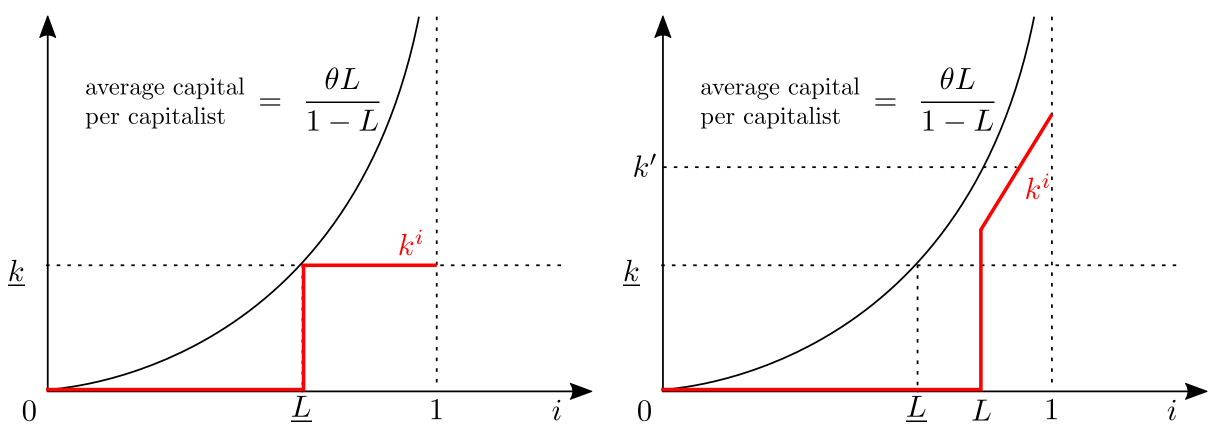
3.2 Welfare in laissez-faire equilibria

While the set of steady states with capital management depends on the management technology p, for any p there exist multiple steady states; it is of interest to know whether these are welfare ranked. Defining welfare as aggregate utility, in any steady state with capital management, social welfare is

$$W = \int \frac{\ln c^i}{1-\beta} di = L \frac{\ln w}{1-\beta} + \int_{1-L}^{1} \frac{\ln[(1-\beta)Rk^i]}{1-\beta} di$$

where without loss of generality we let $[1-L, 1] = \{i : k^i > 0\}$ denote the set of individuals managing capital. All steady states have the same capital-labor ratio θ and prices w, R.

Steady state distributions differ in both the measure of capitalists $1-L$, and the distribution of wealth among capitalists. But given the measure of capitalists, the aggregate wealth of capitalists $K = \theta L$, and their per capita wealth $\dfrac{K}{1-L} = \dfrac{\theta L}{1-L}$, are both pinned down. The higher is the measure of workers L, the fewer capitalists there are, but the richer the average capitalist is. In fact, higher L increases total capital, and total aggregate income $F(\theta, 1)L$. However, the gains from this increase are all accrued by the set of remaining capitalists. Workers earn the same wage w in any steady state, and the displaced capitalists are worse off. In this sense, there is a tradeoff between equity and productive efficiency across steady states. (Note though that none of these steady states are particularly egalitarian; there is always a large measure of workers with zero wealth.)



(a) Most egalitarian steady state

(b) A less egalitarian steady state

Figure 3. Two possible steady states with capital management

Figure 3 depicts two possible steady states. Figure 3a shows the most egalitarian steady state with capital management: all capitalists have the same level of wealth, and this is the smallest possible level, $\underline{k}$. Figure 3b depicts a less equal distribution. This steady state features a higher average level of capital per capitalist, $k' > \underline{k}$, and a correspondingly larger labor force, $L = \frac{k'}{k'+\theta} > \underline{L}$. In addition, not all capitalists have the same level of capital.

We consider the problem of a social planner who solves

$$\max_{\{k^i\},L} L\frac{\ln w}{1-\beta} + \int_{1-L}^{1} \frac{\ln[(1-\beta)Rk^i]}{1-\beta} di$$
$$\text{s.t.} \int_{1-L}^{1} k^i di = \theta L$$
$$k^i \geq \underline{k}(p), \forall i \geq 1-L$$

Clearly, given L, social welfare is maximized when the distribution of wealth among capitalists is equal, $k^i = \dfrac{\theta L}{1-L}$ for all $i \in [1-L, 1]$. For example, the allocation in Figure 3b could be improved by equalizing capital among capitalists. In such cases welfare equals

$$W(L) = \frac{1}{1-\beta}\left\{L \ln w + (1-L)\ln\left(\frac{(1-\beta)R\theta L}{1-L}\right)\right\}$$

which is concave and nonmonotonic in L. Intutively, a utilitarian social planner would like to increase the number of capitalists, because they have higher utility; but she would also like to increase aggregate output, which is decreasing in the number of capitalists. This function has some interior maximand $L^* \in (0,1)$ characterized by the optimality condition

$$\ln\left(\frac{(1-\beta)R\theta L}{1-L}\right) - \ln w = \frac{1}{L}$$

However, L^* may not correspond to a sustainable steady state, because the wealth of each capitalist k must also satisfy $k = \dfrac{\theta L}{1-L} \geq \underline{k}(p)$. Thus the most egalitarian steady state possible, given p, features $L = \dfrac{\underline{k}(p)}{\theta + \underline{k}(p)}$.

Proposition 3. *For p sufficiently low, L^* is attainable. In this case, the welfare-maximizing steady state is not the most egalitarian steady state:*

$$L = L^* > \frac{\theta}{\theta + \underline{k}(p)}$$

For p sufficiently high, L^* is not attainable. In this case, the welfare-maximizing steady state is the most egalitarian one, with

$$L = \frac{\theta}{\theta + \underline{k}(p)} > L^*$$

Proof. See Appendix C. □

Intuitively, there is some unconstrained optimal level of employment L^* which balances the benefit from a higher labor force (richer capitalists) and the cost (fewer capitalists). When the return from part-time management $p(e)$ is sufficiently high, the minimum wealth of each capitalist is relatively high, and the minimum labor force is high, higher than the unconstrained optimum.

When p is relatively low, the minimum wealth of each capitalist is low, the constraint does not bind, and the planner is free to choose the unconstrained optimum. Combining this result with our previous comparative statics result, the following Corollary is immediate.

Corollary 3. *Consider a sequence of functions $\{p_n\}$ with $p_n'(1) \to 0$. As $n \to \infty$, social welfare in the best steady state converges to $\dfrac{\ln w}{1-\beta}$.*

Again, it is striking that as the efficiency of the capital management technology improves, long-run aggregate welfare eventually falls—since in any steady state, welfare is greater than $\ln w/(1-\beta)$, as all individuals have the option of becoming workers.[9]

3.3 Explosive inequality

We have seen that as the marginal return to effort for full time capitalists converges to zero, steady state inequality – the gap between the poorest capitalist and the richest worker – becomes unboundedly large. What if the marginal return to effort for full time capitalists *equals* zero? In this case, no steady state of the kind described above can exist. In any equilibrium where R_t converges to β^{-1}, we find that inequality becomes unboundedly large *along the equilibrium path.*

Proposition 4. *Suppose $p'(1) = 0$ and that $R_t \to \beta^{-1}$ in equilibrium. Then:*

1. *for every $i < 1$, there exists $T(i)$ such that $t > T(i)$ implies $w_t = c_t^i$, $k_t^i = 0$.*

2. *$k_t^1 \to \infty$.*

Proof. See Appendix G. □

Figure 4 illustrates trajectories of capital for selected individuals in such an equilibrium. As the red line indicates, there are some individuals who always have zero capital and work for a wage. Individuals with intermediate levels of capital accumulate wealth for a period of time. After some point however, each individual finds that R_t is so low that, given her position in the wealth distribution, she prefers to decumulate capital. Indeed, for *every* individual except the very richest (for all $i < 1$), there is some date $T(i)$ at which she reaches zero capital and becomes a wage laborer. As time progresses, the set of individuals still managing capital becomes ever smaller, while the mass of propertyless workers converges to 1. At the same time, the aggregate effective capital stock converges to a positive amount consistent with $R_t \to \beta^{-1}$. This implies that the average per capita wealth of the shrinking capitalist class becomes unboundedly large, with the individuals at the top of the distribution soaring away from the rest.

The driving force behind this result is that as the richest individuals accumulate capital, they push down the marginal product of capital, making it increasingly unprofitable for individuals with

[9]These results only pertain to steady state welfare. It is possible that such a change in technology leads to higher aggregate welfare along the transition. However, this is by no means certain, since the associated change in factor prices might redistribute away from individuals with higher marginal utility.

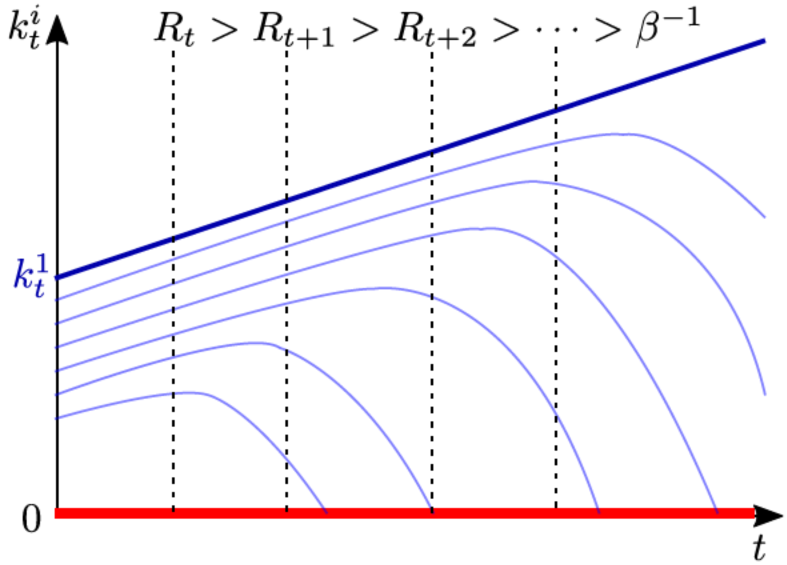


Figure 4. Equilibrium with $p'(1) = 0$ and $R_t \to \beta^{-1}$

a middling level of wealth, who are managing that wealth part-time, to keep accumulating. Thus one by one, these middling capitalists are pushed out by the richest.

4 Egalitarian steady states: existence and fragility

We have seen that capital management can be a powerful force creating long run wealth inequality; moreover, policies and interventions which reduce return differentials can actually increase long-run inequality. How then can inequality be curtailed? Since long-run outcomes in economies with increasing returns can be highly sensitive to initial conditions, one might wonder whether a one-time wealth redistribution can permanently reduce inequality in our economy. This is tantamount to asking whether there exist other steady states with less inequality than those described above.

In this Section, we show that more egalitarian steady states can exist when the average return to effort is not too high. In these steady states, no individuals spend time managing capital. Unlike the steady states which featured capital management, these steady states may be very egalitarian – it is possible for all individuals to have the same wealth.[10] In fact, a certain degree of equality is not just possible but necessary in order for zero-management steady states to exist. If one individual were much richer than her peers, she would grow her wealth and become a capital manager, pushing down the rate of return on capital and disrupting the egalitarian steady state. However, not only are such egalitarian steady states inefficient, we also show that they are unstable: an arbitrarily small measure of rich individuals disrupt the steady state.

[10] In the case of perfect equality, one interpretation is that all individuals are self-employed, in which case this steady state features no wage labor, as well as no capital management.

4.1 Existence

More concretely, consider a putative steady state in which all individuals exert zero effort and roll over their capital, consuming $c^i = w + (Rp(0) - 1)k^i$ each period. In order for these individuals to maintain constant consumption, we need $\beta p(0)R = \beta p(0)F_K(\theta, 1) = 1$. Thus it is already clear that such egalitarian steady states feature a lower effective capital-labor ratio than the unequal steady states described above, and they feature lower output than the most efficient unequal steady states (in which L is close to 1). Under what conditions is this actually a steady state?

Proposition 5. *There exist steady states in which no individuals actively manage capital ($e^i = 0, \forall i$) and $R = (p(0)\beta)^{-1}$. Individuals consume $c^i = (Rp(0) - 1)k^i + w$. For any p, there exists $0 \leq k_{max}(p) < \infty$ such that all individuals must have $k^i < k_{max}(p)$ in such a steady state. A necessary condition for existence is that the average return to capital management is not too high:*

$$p(1) - p(0) < 1 - \exp\{-(1 - \beta)\}.$$

Proof. See Appendix D. □

When such an egalitarian wealth distribution can be sustained as a steady state, a one-time redistribution of wealth which results in such a distribution can *permanently* reduce wealth inequality. For this to work, however, the levelling of wealth must be sufficiently radical that no individual has an incentive to accumulate wealth and become a full-time manager. Steady states without capital management not only permit, but also require, a certain degree of equality. Clearly if an individual has wealth greater than $\hat{k}$ they would rather manage capital full time than work for a wage; but even if they are not rich enough that such static deviations are profitable, they may be tempted by a dynamic deviation. One such dynamic deviation involves consuming less today, accumulating wealth until full time management becomes profitable, and then managing their capital full time. Upon becoming a full-time manager, individuals earn supernormal returns, and enjoy steadily growing consumption forever. In any zero-management steady state, everyone's wealth must be low enough (below k_{max})) that such dynamic deviations are not optimal.

For some parameter values, however, these deviations would increase utility even for an individual with *zero* wealth. In such cases, clearly, no egalitarian steady state can exist, and substantial wealth inequality will re-emerge following any one-time redistribution. This is more likely to be the case when $p(0)$ is relatively small, so that the gap between the returns on managed and unmanaged capital – $\frac{p(1)R}{p(0)R} = \frac{1}{p(0)}$ – is sufficiently large.[11]

[11] Recall that in the unequal steady states described in Section 3, propertyless workers never want to deviate and accumulate capital. The difference is that this egalitarian economy features a lower capital-labor ratio, higher returns to capital, and lower wages, increasing the gains from such deviations.

4.2 Fragility

Even when egalitarian steady states do exist – implying that a one-time redistribution can permanently reduce wealth inequality – these steady states are unstable. Egalitarian steady states require everyone to have only a modest level of wealth, since individuals with a large fortune would be tempted to manage it actively, accumulating further wealth and disrupting the steady state. Thus, if even a small number of rich individuals enter this society, they will accumulate enough wealth to push down the return on capital, inducing the propertied full-time workers to run down their capital stocks and become propertyless workers. To see this, it is useful to make the technical assumption that the labor share of income remains bounded away from zero as capital converges to zero.[12]

Assumption 4. $\lim_{K\to 0} \dfrac{F_L(K,1)}{F(K,1)} > 0$

This implies that $\frac{F_L(\theta,1)}{\theta} \to \infty$ as $\theta \to 0$, since $\frac{F_L(\theta,1)}{\theta} = \dfrac{F_L(\theta,1)}{F(\theta,1)} F\left(1, \frac{1}{\theta}\right)$ and the second term diverges to ∞ as $\theta \to 0$. With this assumption, we can formally show that the introduction of a small number of rich individuals disrupts an egalitarian steady state.

Proposition 6 (Equality is unstable)**.** *Take any egalitarian steady state. There exist $k_s > k_{max}$ and $\bar{\varepsilon} > 0$ such that, for any $\varepsilon \in (0, \bar{\varepsilon})$, if we increase the date 0 wealth of ε individuals to at least k_s, the economy does not converge to an egalitarian steady state.*

Proof. See Appendix E. □

In general, moving from an egalitarian to an inegalitarian steady state has ambiguous effects on aggregate output. On the one hand, the transition increases the capital-labor ratio, which would raise output if there were full employment in both economies. On the other hand, inegalitarian steady states have a lower labor force, as some individuals manage capital full-time instead of working. In the most efficient inegalitarian steady states with $L \to 1$, the first effect dominates, and such inegalitarian steady states dominate egalitarian steady states in efficiency terms.

While moving from an egalitarian to an inegalitarian steady state can improve long-run efficiency, it can also have sizeable effects on long run welfare. Consider for example the perfectly egalitarian steady state, and suppose $p(0)$ is very close to 1 (ensuring that such a steady state exists). In this steady state, each individual's income is equal to per capita GDP. If this steady state is disrupted, and the economy transitions to an inegalitarian steady state of the kind described previously, most individuals will become propertyless workers, earning only the labor share of per capita GDP. If $p(0)$ is close to 1, GDP will be at most slightly higher in any inegalitarian steady state, so this involves a sizeable long run cost for most individuals.

Even if the transition hurts most individuals in the long run, it may benefit all of them in a dynamic sense. Indeed, starting from a perfectly egalitarian steady state, any redistribution of

[12] This is satisfied for Cobb-Douglas production functions.

wealth can only make individuals better off. For one interpretation of the egalitarian steady state is that individuals to work in their own firm and accumulate capital without interacting with the wider world at all; this is optimal given prices in such a steady state. Any change in prices, then, can only introduce gains from trade and increase lifetime utility.[13]

This result is special to the case of a perfectly egalitarian steady state, and does not imply that the introduction of the new rich is Pareto improving in general. Consider an egalitarian steady state where half the population are propertyless workers, and half are propertied workers who do not actively manage their wealth. The entry of the new rich may well reduce the income of the propertied workers, driving up wages and pushing down the return propertied workers can earn on their capital without managing it full time.

Finally, we note that the introduction of the new rich does not require an injection of capital from outside the economy. For example, skimming a small amount off everyone's income and giving it to a small enough class of people would disrupt this equilibrium. Outside our deterministic model, the same situation could arise if individuals faced infrequent but large shocks to their productivity or factor endowments; if some individuals receive large bequests; and so on. Thus transitory shocks to the distribution of wealth can have large, persistent aggregate effects. In a world where this is possible, it would hardly be surprising if individuals had very strong preferences regarding inequality and redistribution.

5 Financial intermediation

One might think that financial intermediation could dampen the forces leading to inequality in our economy, by allowing people with middling amounts of wealth to lend to full-time managers with high wealth and earn higher returns. We now introduce a competitive financial intermediation sector to investigate this hypothesis. In fact, a more efficient financial sector is isomorphic to an upward shift in the $p(e)$ function discussed above – it *increases* the minimum gap between the richest worker and the poorest capital manager.

Competitive intermediaries can borrow d units of output from households at date t and lend these to other households. At date $t+1$, an intermediary collects $R_t^B d$ from its debtors and repays $R_t^A d$ to its creditors. In order to deliver $R_t^A d$ units of output to creditors, however, the intermediary must ship $\psi R_t^A d$ units of output where $\psi > 1$. A lower ψ represents more efficient intermediation. Our implicit assumption in the previous sections was that $\psi = \infty$. The intermediary's profit maximization problem is

$$\max_{d \geq 0} (R_t^B - \psi R_t^A) d$$

Thus we must have $R^B \leq \psi R_t^A$, with strict equality if intermediaries are active.

[13]This result only applies if we interpret our households as infinitely lived individuals. If the household is interpreted as a dynasty, the transition to an inegalitarian steady state would benefit the older generations, who run down the family estate, but it could very well harm the younger generations who have lost their inheritance.

At date t, households can now lend $a_{t+1} \geq 0$ units of output to intermediaries, and they can borrow $b_{t+1} \geq 0$ from intermediaries. The household budget constraint becomes

$$c_t + k_{t+1} + a_{t+1} - b_{t+1} = p(e_t)R_t k_t + R_t^A a_t - R_t^B b_t + w_t(1 - e_t)$$

In equilibrium, the loan market must clear, $\int a^i_{t+1} di = \int b^i_{t+1} di = d$. $x_{t+1} = k_{t+1} + a_{t+1} - b_{t+1}$ represents a household's net saving. For a given level of time spent managing capital e_{t+1}, optimal portfolio decisions solve

$$\tilde{p}_t(e_{t+1})R_{t+1}x = \max_{a,b,k \geq 0} p(e_{t+1})R_{t+1}k + R^A_{t+1}a - R^B_{t+1}b$$
$$\text{s.t. } k + a - b = x$$

A solution to this problem only exists if $R_B \geq R_{t+1}$ – otherwise a household could earn an arbitrarily large leveraged return – and so this must be true in equilibrium. Given that this condition is satisfied, households never strictly prefer to borrow, but they may be indifferent between borrowing and not borrowing (if they plan to exert maximum effort) and so we can replace the individual budget constraint with

$$c_t + x_{t+1} = \tilde{p}_t(e_t)R_t x_t + w_t(1 - e_t)$$

Further, if R_t^A/R_t and R_t^B/R_t are independent of t, $\tilde{p}_t(e)$ will be as well. We have

$$\tilde{p}(e) = \max\left\{p(e), \frac{R_t^A}{R_t}\right\}, \qquad \frac{R_t^A}{R_t} \geq \frac{R_t^B}{\psi R_t} \geq \frac{1}{\psi}$$

Thus the introduction of financial intermediation, or an improvement in its efficiency, is isomorphic to an increase in $p(0)$, or the efficiency with which capital can be managed without exerting effort.

Lemma 5. *Let $\{R^A, R^B, R, w, K, L, a^i, b^i, c^i, e^i, k^i\}$ be a steady state of the economy with cost of financial intermediation ψ and return to effort $p(e)$ in which some individuals exert maximum effort $e^i = 1$. Then there is no intermediation in steady state: $a^i = b^i = 0, \forall i$. Furthermore, $\{R, w, K, L, c^i, e^i, k^i\}$ is also a steady state of the economy without financial intermediation and with return to effort $\hat{p}(e)$ defined as the concave envelope of* $\max\{\psi^{-1}, p(e)\}$*. Conversely, any steady state of the $\hat{p}$ economy in which some individuals exert maximum effort is also a steady state of the economy with intermediation.*

Proof. See Appendix H. □

In particular, financial intermediation does not change the result that the steady state distribution of wealth (in an equilibrium where capital is managed) must feature a gulf between the poorest capitalist and the mass of propertyless workers. In such a steady state, capitalists reinvest their wealth at rate β^{-1}, and do not borrow. Workers have no wealth and thus do not save. Along the transition to such a steady state, workers are able to save at a higher rate than would be possible in the absence of financial intermediation. Nevertheless, in the long run, these individuals face a lower rate of return than full-time capitalists, and so optimally decumulate their wealth.

Like an increase in $p(0)$, a more efficient financial sector (lower $\psi > 1$) increases the minimum possible level of wealth inequality. It follows from Proposition 2 that as intermediation becomes perfectly efficient, $\psi \to 1$, the wealth gap between workers and the poorest capitalist becomes infinitely large, the measure of workers $L \to 1$, and the average wealth of each capitalist becomes infinitely large. In the long run, financial intermediation does not prevent the wealth inequality engendered by capital management; it exacerbates it.[14]

It has been argued that a decline in equity mutual fund fees over the last 50 years played an important role in doubling the percentage of U.S. households owning stocks (Duca, 2005; Duca and Walker, 2022). Over the same period, the share of equity managed by individuals declined (Stambaugh, 2014) and wealth inequality increased. To the extent that falling mutual fund costs can be captured by a decline in ψ in our model, these patterns are consistent with our results: households with middling amounts of wealth to invest give their wealth to intermediaries to manage, and this ultimately leads to a concentration of wealth at the top. To be clear, how significant this mechanism is for explaining the increase in U.S. wealth inequality is an empirical question beyond the scope of this paper. Indeed, despite being able to access the same innovations, other advanced economies have not observed the same increase in wealth inequality seen in the U.S. We only wish to highlight that improving the access of lower wealth households to financial opportunity need not reduce wealth inequality. Indeed, the opposite occurs in our model.

6 Optimal policy

From a normative perspective, capital management introduces a new tradeoff between equity and efficiency. Increasing returns to scale in capital management mean that it is more efficient for a few individuals to manage capital full-time, while the rest of society works, than for everyone to manage capital part time. But since the minority of capital managers necessarily earns a higher rate of return in equilibrium, they will come to own all of society's capital stock in the long run, leading to substantial wealth inequality.

A utilitarian social planner would, if possible, simply instruct a vanishingly small minority of households to manage capital full time and share the proceeds with the workers. Thus an upper bound on long run welfare is $\bar{W} := \frac{\ln(F(\theta,1)-\theta)}{1-\beta}$ where $\beta F_K(\theta, 1) = 1$. This is the level of welfare that would be attained in steady state if a zero measure of individuals managed the capital stock setting $e = 1$, while a unit measure of individuals work full time, and consumption is equalized across individuals.

Whether policy can actually attain this upper bound, or improve on the competitive equilibrium

[14] However, we can not rule out the possibility that more efficient financial intermediation reduces inequality in the short term, or increases the welfare of households with middling wealth who are immiserated in the long run.

at all, depends on both the technology available to policymakers for redistributing resources from managers to non-managers, and the potential deviations available to capital managers. In our baseline model we abstracted from financial intermediation altogether, implying that the private sector could not transfer resources from managers to non-managers; in Section 5, we relaxed this assumption by allowing for costly financial intermediation. It is natural to suppose that the government faces similar transaction costs when transferring from managers to non-managers. We therefore permit transfers $T_t(i,j) \geq 0$ from agent i to j. In order to deliver $T_t(i,j)$ units of the consumption good to j, one must ship $\widetilde{\psi} T_t(i,j)$ goods from i, where $\widetilde{\psi} > 1$, representing the transaction costs associated with redistribution. In particular, we can assume $\widetilde{\psi} \geq \psi$: public redistribution is no more efficient than private financial intermediation. Household budget constraints become

$$c_t^i + k_{t+1}^i = p(e_t^i) R_t k_t + (1 - e^i) w_t + \int [T_t(j,i) - \widetilde{\psi} T_t(i,j)] dj$$

To study how (if at all) outcomes under optimal policy differ from those in competitive equilibrium, we begin by studying a constrained planning problem in a fraction $L < 1$ of households works full time in every period, a fraction $1 - L$ manages capital full time, the managers make transfers to the workers (but not vice versa), and consumption is the same within groups. Letting c_t^L denote the per capita consumption of workers and c_t^H that of managers, the planner's problem can be written

$$\begin{aligned}
\max_{\left\{c_t^L, c_t^H, k_{t+1}, T_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t & \left\{ L \ln c_t^L + (1 - L) \ln c_t^H \right\} \\
c_t^L &= F_L \left(k_t(1 - L), L \right) + (1 - L) T_t \\
c_t^H + k_{t+1} &= F_K \left(k_t(1 - L), L \right) k_t - \widetilde{\psi} L T_t \\
T_t &\geq 0
\end{aligned}$$

Here T_t denotes the transfer sent by each manager $i \in (1 - L, 1]$ to each worker $j \in [0, L]$. Thus the total quantity of resources received as transfers is $(1 - L) L T_t$, and the total amount of resources shipped is $\widetilde{\psi}(1 - L) L T_t$.

As we show in Appendix I, the planner optimally limits the degree of consumption inequality between workers and managers: $c_t^H \leq \widetilde{\psi} c_t^L$. Since the cost of shipping one unit of the consumption good from a manager to the worker is $\widetilde{\psi}$, it can never be optimal to let the gap in marginal utilities become wider than this. Further, it is straightforward to show that the steady state that maximizes welfare features $L \to 1$. The planner would like a vanishingly small mass of individuals to manage the capital stock on society's behalf and transfer almost all their capital income to the workers, who constitute almost all of the population. This could be broadly interpreted as a social wealth fund such as the Alaska Permanent Fund in which capital is publicly owned and managed by a small number of full-time managers who receive a salary $c_t^H - c_t^L + w_t$, with revenues from the fund financing a dividend $c_t^L - w_t$ paid to all households (Meade, 1964; Atkinson, 2015; Bruenig, 2018). Such an arrangement is optimal even if the government faces the same, or higher, costs as the private sector when administering the fund and distributing the benefits.

These outcomes cannot be sustained as laissez-faire equilibria; in fact, they differ markedly from such equilibria. Both steady states feature a minority of individuals managing capital full time and a majority working full time. But under the optimal steady state with transfers, consumption inequality is bounded above, with the gap vanishing as the cost of intermediation goes to zero. In the inegalitarian steady state, consumption and wealth inequality are bounded below, with the gap *diverging* as the cost of intermediation goes to zero. In the long run, only a small minority can benefit from the scale efficiencies associated with increasing returns to scale in capital management under laissez-faire. The solution is not to make everyone a small-scale capital manager, which would be inefficient and fragile, but to exploit scale economies while sharing the benefits.

7 Conclusion

Wealthier individuals have both the motive and the means to earn a higher return on their wealth than poorer individuals. Incorporating these facts into an otherwise standard Ramsey-Cass-Koopmans model generates substantial inequality in the long run. In steady state, the majority of the population works and holds no capital, while a small minority holds all the capital, manages it full time, and does not work. The lower the return to asset management, the wider the gulf between the poorest capitalist and the richest worker in steady state. Similarly, more efficient financial intermediation, which increases the returns available to full-time workers, actually exacerbates long-run wealth inequality. While under certain conditions more egalitarian steady states can exist even under laissez-faire, they are fragile—adding a few wealthy individuals disrupts such a steady state forever—and inefficient, since they do not exploit the scale efficiencies associated with increasing returns to capital management. A utilitarian planner would instead instruct a few individuals to manage capital on behalf of society and transfer most of their income to the workers.

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A Proof of Lemma 3

Suppose $\{\tilde{k}_t^i\}$, $\{\hat{k}_t^i\}$ are both optimal for i. If for some $t \geq 0$ we have $\tilde{k}_{t+1}^i = \hat{k}_{t+1}^i > 0$, then $\tilde{k}_t^i = \hat{k}_t^i$. To see this, note that if $\{\tilde{k}_t^i\}$, $\{\hat{k}_t^i\}$ are optimal, then so is the sequence $\{..., \hat{k}_t^i, \tilde{k}_{t+1}^i, \tilde{k}_{t+2}^i, ...\}$. Both this sequence and $\{\tilde{k}_t^i\}$ satisfy the Euler equation

$$u'(f_t(\hat{k}_t^i) - \tilde{k}_{t+1}^i) = \beta f'_{t+1}(\tilde{k}_{t+1}^i) u'(f_{t+1}(\tilde{k}_{t+1}^i) - \tilde{k}_{t+2}^i)$$
$$u'(f_t(\tilde{k}_t^i) - \tilde{k}_{t+1}^i) = \beta f'_{t+1}(\tilde{k}_{t+1}^i) u'(f_{t+1}(\tilde{k}_{t+1}^i) - \tilde{k}_{t+2}^i)$$

Thus, $\hat{k}_t^i = \tilde{k}_t^i$. We now prove the main result. We must have

$$u(f_t(k_t^h) - k_{t+1}^h) + \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^h) \geq u(f_t(k_t^h) - k_{t+1}^l) + \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^l)$$
$$u(f_t(k_t^l) - k_{t+1}^l) + \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^l) \geq u(f_t(k_t^l) - k_{t+1}^h) + \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^h)$$

Summing,

$$u(f_t(k_t^h) - k_{t+1}^h) + u(f_t(k_t^l) - k_{t+1}^l) \geq u(f_t(k_t^h) - k_{t+1}^l) + u(f_t(k_t^l) - k_{t+1}^h)$$
$$u(f_t(k_t^h) - k_{t+1}^h) - u(f_t(k_t^l) - k_{t+1}^h) \geq u(f_t(k_t^h) - k_{t+1}^l) - u(f_t(k_t^l) - k_{t+1}^l)$$
$$g(k_{t+1}^h) \geq g(k_{t+1}^l)$$

where $g(x) := u(f_t(k_t^h) - x) - u(f_t(k_t^l) - x)$ is increasing. Thus we have $k_{t+1}^h \geq k_{t+1}^l$. From the first result above, it follows that $k_{t+1}^h > k_{t+1}^l$ unless $k_{t+1}^h = k_{t+1}^l = 0$.

B Proof of Proposition 1

The proof has a number of steps. First we show that if $\beta R = 1$, individuals with zero capital act in the way described. Next, we show that there exists $\underline{k}$ such that individuals deviate from the strategy described if $k < \underline{k}$. We then present upper and lower bounds on $\underline{k}$. Finally, we show existence by explicitly constructing a steady state.

Lemma 6. *If $\beta R_t = 1$ for all t, an individual with zero capital stays at zero forever.*

Proof. By Corollary 2, if an individual with $k_0 = 0$ chooses $k_1 > 0$, capital must be increasing (and thus positive) forever, so the Euler equation always holds with equality. Then consumption is nonincreasing for this individual, from the Euler equation with $\beta R = 1$. Furthermore, $c_0 < w$ from the budget constraint. So $c_t < w$ for all t. This is not optimal, since the deviation $c_t = w$ for all t is feasible and yields higher utility. $\square$

Having shown that individuals with $k = 0$ do not wish to deviate from the strategies we have given them, we ask when individuals with positive wealth $k > 0$ would like to deviate from the 'Mertonian' strategy $c = (1-\beta)Rk$. The following Corollary will be useful.

Corollary 4. *If $\beta R_t = 1$ for all t, consumption is nonincreasing for all individuals.*

Proof. If an individual has positive capital at $t+1$, the Euler equation stipulates that $\frac{c_{t+1}}{c_t} = \beta R p(e_{t+1}) \leq 1$. If the individual has zero capital at date $t+1$, then $c_{t+1} = w$, since (by monotonic trajectories) it is optimal to stay at zero forever. If $c_{t+s} = w$ for all $s \geq 1$, it cannot be optimal to have $c_t < w$, then the deviation $c_t = w$ for all t is feasible and preferable. So even if $k_{t+1} = 0$, capital must be nonincreasing between t and $t+1$. $\square$

It follows that starting from some level of wealth $k_0 > 0$, it cannot be optimal to choose $k_1 > k_0$ and consume $c_0 < (1-\beta)Rk_0$, since that would necessarily involve a lower level of consumption (at every date t) than in the Mertonian plan. Thus the only deviations to consider are those which involve $k_1 < k_0$. It follows immediately from Lemma 3 that if an individual with k_0 chooses $k_1 < k_0$, an individual with $k_0' < k_0$ also chooses $k_1' < k_0'$; conversely, if an individual with k_0 chooses $k_1 = k_0$, an individual with $k_0'' > k_0$ also chooses $k_1'' = k_0''$. Thus there must exist some critical level of wealth $\underline{k}$ - possibly 0 or infinity, at this stage - such that individuals with $k_0 < \underline{k}$ decumulate wealth, and those with $k_0 > \underline{k}$ maintain that level of wealth.

It is straightforward to show that $\underline{k} \geq \hat{k}(R, w)$. If $k_0 < \hat{k}$, an individual rolling over this level of capital would find it optimal to exert less than full effort, earning a return of less than β^{-1}, thus their consumption would be declining over time. It only remains to show that $\underline{k} \leq \tilde{k}$, i.e. that an individual with $k_0 > \tilde{k}$ strictly prefers not to deviate.

Suppose by contradiction that $k_0 > \tilde{k}$ and it is optimal to set $c_0 > (1-\beta)Rk_0$. Let T be the first date at which $k_T < \hat{k}$. Either $k_T > 0$, or $k_T = 0$. If $k_T > 0$, then an Euler equation holds between $T-1$ and T. So we must have $\frac{c_T}{c_{T-1}} = \beta R p(e_T) > p(0)$. However, we have

$$
\begin{aligned}
&c_{T-1} > (1-\beta)Rk_0 > (1-\beta)R\tilde{k} \geq (1-\beta)R\frac{\hat{k}}{p(0)(1-\beta)} = \frac{\hat{k}}{p(0)} \\
&c_T \leq f(k_T) \leq f(\hat{k}) = R\hat{k}, \text{ which implies} \\
&\frac{c_T}{c_{T-1}} < \frac{R\hat{k}}{R\hat{k}/p(0)} = p(0),
\end{aligned}
$$

a contradiction. Suppose then that $k_T = 0$; then $k_t = 0$ for all $t > T$. The continuation utility from this strategy, starting at $T-1$, is $\ln(Rk_{T-1}) + \frac{\beta}{1-\beta}\ln w$. If instead the individual deviates and pursues a Mertonian strategy starting at $T-1$, this yields $\frac{\ln((1-\beta)Rk_{T-1})}{1-\beta}$. The gain from this deviation is

$$
\frac{\ln(1-\beta)}{1-\beta} + \frac{\beta}{1-\beta}[\ln(Rk_{T-1}) - \ln w] > \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta}{1-\beta}\left[\ln(k_{T-1}) - \ln(p'(1)\hat{k})\right]
$$

This will be positive if $k_{T-1} > p'(1)\hat{k}(1-\beta)^{-1/\beta}$. But we have $k_{T-1} = \dfrac{c_{T-1}}{R} > \dfrac{(1-\beta)Rk_0}{R} > (1-\beta)\tilde{k} \geq p'(1)(1-\beta)^{-1/\beta}\hat{k}$, so the gain is indeed positive, and the putative strategy cannot be optimal. So, no deviation from a Mertonian strategy can be optimal when $k_0 > \tilde{k}$, and this strategy is indeed optimal.

Since for all individuals with capital $k^i \geq \hat{k}$, we have $\int k^i di \geq \hat{k}(1-L)$. Then by definition, we have $\theta = \frac{\int k^i di}{L} \geq \frac{\hat{k}(1-L)}{L}$, as claimed. To show existence, it suffices to construct a steady state where all capitalists have wealth $\tilde{k}$. We need $\beta F_K(\tilde{k}(1-L), L) = 1$. The left hand side goes to 0 as $L \to 0$ and goes to ∞ as $L \to 1$; by continuity, there exists $L \in (0,1)$ such that the condition is satisfied.

C Proof of Proposition 3

The second part is immediate, since $L^* < 1$ and $\dfrac{\theta}{\theta + \underline{k}(p)} \to 1$ as $p'(1) \to 0$. To see the first part, set $p(e) = e$. In this case we use the following Lemma.

Lemma 7. *Suppose $p(e) = e$. Then $\underline{k} = \dfrac{\exp\{1\}w}{R-1}$.*

Proof. By definition, $\underline{k}$ is the smallest level of capital such that no deviation from a Mertonian strategy is optimal. Suppose that, given k_0, a T-period deviation is profitable. Given $p(e)$, it will never be strictly optimal to exert $e \in (0,1)$, so we can restrict attention to corner solutions. If T is the first date at which $e_t = 1$, we must have $k_T = 0$. From the Euler equation, consumption will be constant and equal to c_0 while $e_t = 1$, before date t. From the budget constraint, this gives us $0 = k_T = R^T k_0 - \dfrac{R^T - 1}{R-1} c_0$, i.e. $c_0 = \dfrac{(R-1)k_0}{1-\beta^T}$, given that $R = \beta^{-1}$. Utility from this deviation is

$$\frac{1-\beta^T}{1-\beta} \ln\left(\frac{(R-1)k_0}{1-\beta^T}\right) + \frac{\beta^T}{1-\beta} \ln w = W(\tau) := \frac{\tau}{1-\beta} \ln\left(\frac{Rk_0}{\tau}\right) + \frac{1-\tau}{1-\beta} \ln w$$

where we define $\tau = 1 - \beta^T \in [0,1]$. Note that $\tau = 1$ corresponds to $T = \infty$, i.e. remaining a capitalist pursuing a Mertonian strategy forever. We want to know when this is optimal. Consider a relaxed problem in which the individual can choose τ without integer constraints. Since W is concave in τ, first order conditions are sufficient for an optimum. This first order condition yields

$$\frac{1}{1-\beta}\left[\ln\left(\frac{Rk_0}{\tau}\right) - \ln w - 1\right] = 0 \Rightarrow \ln \tau = \ln\left(\frac{(R-1)k_0}{w}\right) - 1$$

Setting $\tau = 1$, we have $k_0 = \dfrac{ew}{R-1}$ where $e = \exp\{1\}$ denotes Euler's constant. If k_0 is higher than this value, clearly setting $T = \infty$ is superior to any non-Mertonian deviation. If k_0 is less than this value, the optimal $\tau < 1$, and there exists an integer T high enough that $W(1-\beta^T) > W(1)$, i.e. it is optimal to deviate from a Mertonian strategy. Thus as claimed, we have $\underline{k} = \frac{ew}{R-1}$. □

Corollary 5. *For any p, $\underline{k} \geq \dfrac{\exp\{1\}w}{R-1}$.*

$\underline{k} = \dfrac{\exp\{1\}w}{R-1}$ corresponds to $L = \dfrac{\exp\{1\}w}{\theta(R-1)+\exp\{1\}w}$. Taking first order conditions of the welfare function and substituting in this value, we have

$$W'(L) = \frac{1}{1-\beta}\left\{\ln w - \ln\left(\frac{(1-\beta)R\theta L}{1-L}\right) + \frac{1}{L}\right\} = \frac{1}{1-\beta}\left\{-1+\frac{1}{L}\right\} > 0,$$

i.e. starting from the lowest possible L, the planner would prefer a higher L, as claimed. □

D Proof of Proposition 5

First, we prove such steady states cannot exist when $p(0) < p_{dev} := \exp\{-(1-\beta)\}$. Consider deviations of the following form. Starting from $k_0 \geq 0$, an individual chooses consumption $c_0 < w+(Rp(0)-1)k_0$. She keeps consumption at this level until date $T-1$, $c_t = c_0$ for $t \leq T-1$. At date $T-1$, the individual has income $f(k_{T-1}) = Rp(0)k_{T-1}+w$. Starting from this date, the individual manages her capital full time and pursues a Mertonian saving strategy, investing $k_{t+1} = \beta f(k_t)$ and consuming $c_t = (1-\beta)f(k_t)$, so that wealth evolves according to $k_{t+1} = \beta R k_t = k_t/p(0)$, and consumption grows according to $c_{t+1} = c_t/p(0)$. From the budget constraint,

$$k_{T-1} = (Rp(0))^{T-1}k_0 + \frac{(Rp(0))^{T-1}-1}{Rp(0)-1}(w-c_{T-1})$$

since consumption equals c_{T-1} for all $t < T$. For income to be $\frac{1}{1-\beta}$ times consumption, we need $Rp(0)k_{T-1}+w = \frac{c_{T-1}}{1-\beta}$. Combining and using the fact that $\beta Rp(0) = 1$, we obtain $\tau := 1-\beta^T = \frac{c_{T-1}-(\beta^{-1}-1)k_0}{w}$ where $\tau \in [0,1]$ is an increasing transformation of T. Intuitively, it takes longer to reach the point where a Mertonian strategy is viable if either consumption is higher during the transition, or initial wealth is lower. Equivalently, $c_{T-1} = \tau w + (Rp(0)-1)k_0$. When $\tau = 1$, consumption is the same as in the putative steady state, and it takes infinitely long to reach the point where a Mertonian strategy is viable (i.e. this point is never reached, and the individual does not move). We want to see whether $\tau < 1$ is optimal. The utility from this deviation is

$$\begin{aligned} W(\tau) &= \frac{\beta^T}{1-\beta}\ln c_{T-1} + \sum_{t=T}^{\infty}\beta^t \ln\left(p(0)^{-(t-T+1)}c\right) = \frac{\ln c_{T-1}}{1-\beta} + \frac{\beta^T}{(1-\beta)^2}\ln\left(\frac{1}{p(0)}\right) \\ &= \frac{1}{1-\beta}\left\{\ln\left(\tau w + (Rp(0)-1)k_0\right) + \frac{1-\tau}{1-\beta}\ln\left(\frac{1}{p(0)}\right)\right\} \end{aligned}$$

Consider a relaxed problem where the individual chooses $\tau \leq 1$ without integer constraints. The first order condition (which is sufficient for optimality since W is concave) is

$$W'(\tau) = \frac{1}{1-\beta}\left\{\frac{w}{\tau w + (Rp(0)-1)k_0} - \frac{1}{1-\beta}\ln\left(\frac{1}{p(0)}\right)\right\} \geq 0, \tau \leq 1, \text{ at least one equality}$$

At $\tau = 1$, we have $W'(1) < 0$: thus $\tau = 1$ is not optimal in the relaxed problem, whenever

$$\left\{ \frac{w}{w + (Rp(0) - 1)k_0} - \frac{1}{1-\beta} \ln\left(\frac{1}{p(0)}\right) \right\} < 0$$

$$\frac{1}{1 + (\beta^{-1} - 1)k_0/w} - \frac{1}{1-\beta} \ln\left(\frac{1}{p(0)}\right) < 0 \Rightarrow p(0) < \exp\left\{ \frac{-1}{\frac{1}{1-\beta} + \frac{k_0}{\beta w}} \right\}$$

It follows that if $k^i > -\frac{\beta w}{\ln p(0)} - \frac{\beta w}{1-\beta}$, this is indeed an improving deviation. So there exists $k_{max} \leq -\frac{\beta w}{\ln p(0)} - \frac{\beta w}{1-\beta}$ such that if an individual has $k^i > k_{max}$, she has *some* improving deviation, so such a level of wealth is not permissible in a steady state of the kind described.

If the upper bound on k_{max} just described is negative, clearly no steady state of the kind described can exist. Since $\exp\left\{ \frac{-1}{\frac{1}{1-\beta} + \frac{k_0}{\beta w}} \right\} \in (p_{dev}, 1)$, whenever $p(0) < p_{dev}$, we can find T large enough that $\tau = 1 - \beta^T$ is close enough to 1 to ensure that $W(\tau) > W(1)$, and thus we have an improving deviation. It follows that in this case, remaining at $c^i = w + (Rp(0) - 1)k^i$ is not optimal.

Whenever $p(0) \geq p_{dev}$, this particular deviation cannot be optimal. We now construct an example in which *no* deviation can be optimal, namely when $p(e)$ is linear, $p(e) = p(0) + (1 - p(0))e$. To see this, first note that when $\beta p(0) R_t = 1$ for all t, consumption is nondecreasing for all individuals, since $\frac{c_{t+1}}{c_t} \geq \beta R p(e_{t+1}) \geq 1$. Consider a deviation $c_0' > c_0 = (\beta^{-1} - 1)k_0 + w$. Since consumption is nondecreasing, if this deviation is optimal, the individual must consume more than c_0 in every subsequent period. Clearly this eventually violates the individual's budget constraints, a contradiction. Thus it suffices to consider deviations in which $c_0' < c_0$ and $k_1' > k_0$. Since optimal trajectories are monotonic, such a path must feature increasing capital for all t. Until some date T, we will have $k_t < \hat{k} = \frac{w}{R(1-p(0))}$, and consumption satisfies the Euler equation $c_{t+1}/c_t = \beta R p(0) = 1$; after date T, the individual will manage capital full time, and consumption will be growing at rate $c_{t+1}/c_t = 1/p(0) > 1$. That is, any optimal deviation must have the form described above. None of these deviations will be optimal for an individual with initial wealth k_0 if $p(0) \geq \exp\left\{ \frac{-1}{\frac{1}{1-\beta} + \frac{k_0}{\beta w}} \right\}$.

Consider steady states in which everyone has the same level of capital $k_0 = \theta/p(0)$ where θ is defined by $\beta p(0) F_K(\theta, 1) = 1$. Define $H(p(0)) = p(0) - \exp\left\{ \frac{-1}{\frac{1}{1-\beta} + \frac{\theta/p(0)}{\beta w}} \right\}$. As $p(0) \to 1$, $H(p(0)) \to 1 - \exp\left\{ \frac{-1}{\frac{1}{1-\beta} + \frac{\theta}{\beta w}} \right\} > 0$. Thus for $p(0)$ close enough to 1, the condition above is satisfied, and no deviation can be optimal. In this case, an egalitarian steady state exists. □

E Proof of Proposition 6

To prove this Proposition, we need the following three Lemmas.

Lemma 8 (Mertonian bound). *Suppose $\beta R_t \geq 1$ for all t, and suppose some individual never has zero capital, $k_t > 0$ for all t. Then his consumption is at least as high as that of a Mertonian household with the same income: $c_t > (1-\beta) f(k_t)$, for all t.*

Proof. Define a Mertonian path by $c_0^M = (1-\beta) f(k_0)$, $c_{t+1}^M = \beta R_{t+1} c_t^M$. Such a path is feasible (setting $e_t = 1$ for all t). Suppose by contradiction that the optimal path has $c_0 < c_0^M$. From the Euler equation (with holds with equality since, by hypothesis, the individual never has zero capital), we have $c_{t+1} = \beta R_{t+1} p(e_{t+1}) c_t < \beta R_{t+1} c_t$. Thus $c_t < c_t^M$ for all t. This cannot be optimal, since the individual could deviate to a Mertonian plan yielding higher consumption in every period. $\square$

Lemma 9. *If $k_{t+1} = 0$ for some individual, $f(k_t) \leq \dfrac{1}{\beta p(0)} \dfrac{w_{t+1}}{R_{t+1}}$.*

Proof. Since $c_t = f(k_t)$ if $k_{t+1} = 0$, the Euler equation implies $\dfrac{1}{f(k_t)} \geq \dfrac{\beta R_{t+1} p(0)}{c_{t+1}} \geq \dfrac{\beta p(0) R_{t+1}}{w_{t+1}}$. $\square$

Lemma 10. *There exists $\underline{\theta} > 0$ such that if $\theta_{t-1} \geq \underline{\theta}$, $\theta_t \geq \underline{\theta}$.*

Proof. Fix $\underline{\theta}$. For any value of $\underline{\theta}$, we suppose that $\theta_{t-1} \geq \underline{\theta}$, $\theta_t < \underline{\theta}$. First we show that there exists $a > 0$ such that when θ_t is sufficiently low, individuals save at least $a w_{t-1}$. Set $a < (1+\beta)^{-\frac{1+\beta}{\beta}} \beta < \dfrac{\beta}{1+\beta}$. First, take an individual with $k_{t-1} = 0$. Suppose she saves $k_t < a w_{t-1}$, and consider a deviation to $k_t = \beta w_{t-1}/(1+\beta)$, keeping future decisions the same. The gain from this deviation is

$$
\begin{aligned}
&\ln\left(w_{t-1} - \frac{\beta w_{t-1}}{1+\beta}\right) + \beta \ln\left(f_t\left(\frac{\beta w_{t-1}}{1+\beta}\right) - k_{t+1}\right) - \ln\left(w_{t-1} - k_t\right) - \beta \ln\left(f_t\left(k_t\right) - k_{t+1}\right) \\
&\geq \ln\left(\frac{w_{t-1}}{1+\beta}\right) + \beta \ln\left(f_t\left(\frac{\beta w_{t-1}}{1+\beta}\right)\right) - \ln w_{t-1} - \beta \ln\left(f_t\left(k_t\right)\right) \\
&\geq \ln\left(\frac{w_{t-1}}{1+\beta}\right) + \beta \ln\left(R_t \frac{\beta w_{t-1}}{1+\beta}\right) - \ln w_{t-1} - \beta \ln\left(R_t a w_{t-1} + w_t\right) \\
&\geq \ln\left(\frac{w_{t-1}}{1+\beta}\right) + \beta \ln\left(R_t \frac{\beta w_{t-1}}{1+\beta}\right) - \ln w_{t-1} - \beta \ln\left(R_t a w_{t-1} + w_{t-1}\right) \\
&= -(1+\beta)\ln(1+\beta) + \beta \ln \beta - \beta \ln\left(a + R_t^{-1}\right)
\end{aligned}
$$

When $R_t^{-1} = 0$, this expression is positive; so it is positive for R_t sufficiently large, or (equivalently) for θ_t sufficiently small. In other words, there exists some level of θ_t, say $\underline{\theta}(a)$, such that no individuals with $k_{t-1} = 0$ save less than $a w_{t-1}$ when $\theta_t < \underline{\theta}(a)$. By sorting, all individuals save at least $a w_{t-1}$. If $\theta_{t-1} > \underline{\theta}(a)$, then $w_{t-1} > w(\underline{\theta}(a))$, so all individuals save at least $a w(\underline{\theta}(a))$.

Next, fix any $\bar{e}$ close to 1. An individual with capital $a w(\theta)$ will spend at least $\bar{e}$ units of time managing capital when $p'(\bar{e}) R(\theta) a w(\theta) > w(\theta)$, i.e. $p'(\bar{e}) R(\theta) a > 1$. Define $\underline{\theta}(\bar{e})$ as the value of θ such that $p'(\bar{e}) R(\underline{\theta}(\bar{e})) a = 1$. Set $\underline{\theta} < \min\{\underline{\theta}(a), \underline{\theta}(\bar{e})\}$. Then the effective capital-labor ratio satisfies

$$
\theta_t = \frac{\int p(e^i) k_t^i di}{\int (1 - e_t^i) di} > \frac{p(\bar{e}) a w(\underline{\theta})}{1 - \bar{e}}
$$

By Assumption 4, we have $\lim_{\theta \to 0} \dfrac{w(\theta)}{\theta} = \infty$. So there exists $\underline{\theta}$ small enough that, if aggregate prices were consistent with $\theta_{t-1} \geq \underline{\theta}$, $\theta_t < \overline{\theta}$, individuals would choose to save enough capital that the date t capital-labor ratio would in fact be greater than $\underline{\theta}$. $\square$

We are now ready to prove the proposition.

Proof. Call the individuals with $k_0^i \geq k_s$ the 'new rich'. First, note that if (by contradiction) we return to an egalitarian steady state, the new rich can never all have zero capital. If by contradiction they did have zero capital in some period, then by the sorting results above, all households have zero capital, and thus the economy has zero capital forever, and fails to return to steady state. It follows from Lemma 8 that the new rich have date zero consumption at least equal to $(1-\beta)R_0 k_0^i$. Other individuals, meanwhile, have consumption at most $R_0 k_{max} + w_0$.

Next, suppose by contradiction that we do return to an egalitarian steady state. Then the consumption of the new rich must converge to (at most) the consumption of individuals with k_{max} in this steady state - call it c_{max} - while the consumption of all other individuals must converge to (at least) the steady state wage. Choose k_s such that $\frac{(1-\beta)R_0 k_s}{R_0 k_{max}+w_0} > \frac{c_{max}}{w}$. Then consumption ratios must decline over time. If there is even one individual outside the new rich who never has zero capital, this cannot be the case: if another individual j has positive capital for every date $t < T$, then we must have $\frac{c_t^{nr}}{c_t^j} > \frac{(1-\beta)R_0 k_s}{R_0 k_{max}+w_0}$ for all $t < T$. So every individual outside the new rich must pass through zero at some point. Furthermore, they must all do so at the same point, by a straightforward application of the sorting Lemma. Let T be the first date at which everyone except the new rich has zero capital. Suppose by contradiction that at this date, $\frac{c_T^{nr}}{c_T^j} < \frac{(1-\beta)R_0 k_s}{R_0 k_{max}+w_0}$. Since $c_T^j \leq w_T$, this means that the consumption of the new rich must be low, relative to the wage. By the Mertonian bound argument above, this in turn means that their per capita wealth must be low. Thus the aggregate capital labor ratio must be extremely low. In fact, if $\bar{\varepsilon}$ is sufficiently small, the aggregate capital labor ratio must be less than $\underline{\theta}$, a contradiction. $\square$

F Alternative microfoundations for $p(e)$

F.1 Effort in advance

In our benchmark model, households spend time to get a higher return on their capital in the same period that they actually receive capital income. They do not spend time allocating investments at the time when they sacrifice consumption in order to save. This arguably makes it hard to interpret the model as one where households research which financial assets deliver the highest rate of return: households spend time studying the prospective returns on various assets when they choose portfolios, not when the returns are realized. To address this concern, we now consider a variant of the model where households spend time managing capital in the period when they save.

A household who saves a positive amount at date t also chooses how much time $e_t \in [0,1]$ to spend allocating this saving, supplying her remaining units $1-e_t$ of time as a wage laborer. Spending more time allocating savings increases the productivity of capital: s units of output saved, managed with intensity e, generate $k = p(e)s$ units of effective capital. Formally, households solve

$$\max_{\{c_t,k_{t+1},e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t$$

$$\begin{aligned} \text{s.t. } & k_{t+1} = p(e_t)[R_t k_t + w_t(1-e_t) - c_t] \\ & e_t \in [0,1] \\ & k_t \geq 0, \forall t \\ & k_0 \text{ given} \end{aligned}$$

Equivalently, we can write the constraint as $c_t + \left(\frac{k_{t+1}}{p(e_t)} + w_t e_t\right) = R_t k_t + w_t$. A household will choose e_t to solve the static minimization problem $h_t(k_{t+1}) = \min_{e\in[0,1]} \frac{k_{t+1}}{p(e_t)} + w_t e_t$, immediately implying that $h_t(k_{t+1})$ is a concave and increasing function. It is clear that the optimal choice of e_t is (at least weakly) decreasing in w_t and increasing in k_{t+1}. A household who plans to accumulate more effective capital k_{t+1} has a stronger incentive to pay a fixed cost, in terms of time, in order to reduce the amount of output that must be sacrificed to acquire k_{t+1}. However, households will be less willing to pay this fixed cost when their opportunity cost of time is higher.

$h_t(k_{t+1})$ represents the total resources foregone in order to acquire effective capital k_{t+1}. These resources include both foregone output $\frac{k_{t+1}}{p(e_t)}$ and time e_t, where the latter is valued at its opportunity cost (the real wage). Households optimally allocate their time between labor and managing savings in order to minimize the cost of acquiring a given amount of capital. This yields the constraint

$$c_t + h_t(k_{t+1}) = R_t k_t + w_t$$

which is reminiscent of the production possibilities frontier in two sector growth models, except that here the frontier is non-convex. Equivalently, define

$$f_{t+1}(x_{t+1}) = R_{t+1} h_t^{-1}(x_{t+1}) + w_{t+1}$$

to be the capital and labor income one earns at date $t+1$ as a result of forgoing total resources $x_{t+1} := h_t(k_{t+1})$ at date t, assuming that these resources are allocated so as to minimize cost. Note that this represents the 'notional' or total labor income a household would earn if they worked full time at date $t+1$. One interpretation is that households always work full time, but they sometimes hire an individual (at the market wage) to spend $e_t \in [0,1]$ units of time managing their capital.

We can therefore rewrite the individual decision problem as

$$\begin{aligned}
\max_{\{c_t, x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t \\
\text{s.t. } x_{t+1} + c_t = f_t(x_t) \\
x_t \geq 0, \forall t \\
x_0 \text{ given}
\end{aligned}$$

Since h_t is concave, f_{t+1} is convex. Thus this problem has the same form as the model in the main text where effort is chosen at the time of production. Further, since $h_t'(k_{t+1}) = \frac{1}{p(e_t^*)}$ by the Envelope Theorem, $f_{t+1}'(x_{t+1}) = p(e_t^*)R_{t+1}$. Thus the individual Euler equation has a very similar form – $\frac{c_{t+1}}{c_t} \geq \beta p(e_t^*)R_{t+1}$ – except that now it is effort in the first period, not the second, which affects the marginal return to capital. Finally, here $f_t(\cdot)$ depends on the wage at date $t-1$ as well as factor prices at date t. This is because $f_t(x_t)$ measures the amount that can be earned at date t having sacrificed real resources x_t at date $t-1$. If the wage was very high at date $t-1$, in general the same sacrifice of real resources will produce less effective capital, because less 'asset management time' can be purchased for the same sacrifice of output. Thus in general equilibrium, f_t depends on capital-output ratios at both dates $t-1$ and t. In steady state, this difference vanishes, and the model with effort in advance is qualitatively identical to the model in the main text.

F.2 Searching for investment opportunities

There is a set A_t of investment opportunities at time t called industries. Each industry $a \in A_t$ has productivity $z(a) \leq 1$ and a constant returns to scale technology $y = F(z(a)k, l)$ given raw capital k and labor l. Firms are competitive within each industry. A firm in industry a makes profits

$$\pi(a, k) = \max_l F(z(a)k, l) - wl = z(a)F_K(z(a)k/l, 1)k$$

where the effective capital-labor ratio $z(a)k/l$ is common across all firms and industries. Firms raise capital exclusively by selling equity at price $q(a)$ per share. Normalizing the number of shares sold by the representative firm in each industry to unity, the return per share is

$$\frac{\pi(a, k)}{q(a)} = \frac{z(a)F_K(K_{t+1}, L_{t+1})q(a)}{q(a)} = z(a)R_{t+1}.$$

Households are not aware of all industries at the beginnning of time, but must spend time e to become aware of (or evaluate) a subset of these industries $A_t(e)$. The number of industries discovered is increasing in time spent searching: $A_t(e) \subset A_t(e') \subset A_t(1) \subseteq A_t$ for $e < e' < 1$. Assume that while the set of investment opportunities changes each period, the productivity of the best industry a household finds, given e, remains the same: $\max_{a \in A_t(e)} z(a) = \max_{a \in A_\tau(e)} z(a)$ for all τ, t and

$e \in [0,1]$. Then the rate of return that a household earns per unit capital given effort e as $p(e)R_{t+1}$ where $p(e) = \max_{a \in A_t(e)} z(a)$. This is equivalent to the reduced form effort in advance model.

G Proof of Proposition 4

The proof of Proposition 4 has a number of parts. First, we show that individuals' ranks in the wealth distribution do not change over time. Second, we show that the measure of individuals whose consumption is increasing goes to zero. Third, we show that wealth is bounded for almost all individuals. Fourth, we show that for almost all individuals, their effective return on capital is bounded. Finally, we can prove the main result.

First, note that $R_t \to \beta^{-1}$ if and only $\theta \to \theta^*$ defined by $\beta F_K(\theta^*, 1) = 1$.

Lemma 11. *If $\theta_t \to \theta^* < \infty$, then the measure of individuals whose consumption is increasing, $CI_t := \int_0^1 \mathbb{I}\{c_t^i > c_{t-1}^i\} di$, converges to zero.*

Proof. For any date t, define k_t^C by $1 = \beta f_t'(k_t^C) = 1$. Then $c_t^i > c_{t+1}^i$ iff $k_t > k_t^C$, and so $CI_t = \int_0^1 \mathbb{I}\{k_t^i > k_t^C\} di$. First, we show that $k_t^C \to \infty$. To see this, note that $f'(k_t^i) = p(e_t(k_t^i)) F_K(K_t, L_t)$. Since $F_K \to \beta^{-1}$, we must have $p(e_t(k_t^C)) \to 1$, i.e. $k_t^C \to \infty$.

Define $K_t^R = \int k_t^i di$ to be the aggregate stock of raw capital. Note that since $\theta_t \to \theta^* < \infty$, $K_t = \theta_t L_t$ and $L_t \in [0,1]$, K_t is bounded above. Since

$$K_t = \int p(e_t^i) k_t^i di \geq \int p(0) k_t^i di = p(0) K_t^R$$

and K_t is bounded above, we cannot have $K_t^R \to \infty$. But we also have

$$K_t^R = \int \mathbb{I}\{k_t^i \leq k_t^C\} k_t^i di + \int \mathbb{I}\{k_t^i > k_t^C\} k_t^i di \geq 0 + k_t^C \int_0^1 \mathbb{I}\{k_t^i > k_t^C\} di = k_t^C CI_t$$

Since $k_t^C \to \infty$, we must have $CI_t \to 0$. $\square$

Lemma 12 (Eventually No Escape From Zero)**.** *Suppose that $\forall t \geq 0$ and for $\varepsilon > 0$, $R_t \in (\underline{R}, \overline{R})$, $w_t \in (\underline{w}, \overline{w})$, where $\underline{R} = \beta^{-1} - \varepsilon$, $\overline{R} = \beta^{-1} + \varepsilon$, $\underline{w} = w^* - \varepsilon$, $\overline{w} = w^* + \varepsilon$, $w^* = F_L(K^*, 1)$. Assume that for some individual, k_0. For ε sufficiently close to 0, we have $k_t = 0$, $c_t = w_t$ for all t.*

Proof. Suppose by contradiction that $k_t > 0$ for $0 < t < T$ and $k_T = 0$. We allow $T = \infty$, in which case $k_t > 0$ for all $t > 0$. In what follows, let $e(R, w, k)$ denote the solution to

$$\max_e p(e) Rk + (1-e) w$$

and note that e is increasing in R and k and decreasing in w.

We start by observing that $k_1 < w_t < \overline{w}$. Consequently, we have $e_1 < e(\overline{R}, \underline{w}, \overline{w})$, and

$$\frac{c_1}{c_0} = \beta p(e_1) R_1 < \beta p(e(\overline{R}, \underline{w}, \overline{w}))\overline{R} := g(\varepsilon)$$

As $\varepsilon \to 0$, $g(\varepsilon) \to g(0) = p(e(\beta^{-1}, w^*, w^*)) < p(e(\beta^{-1}, w^*, \infty)) = 1$. Since $c_0 < w_0 < \overline{w}$, we have $c_1 < g(\varepsilon)\overline{w}$. Finally, from the individual's Euler equation between $t-1$ and t for any $1 < t < T$:

$$\frac{c_t}{c_{t-1}} = \beta R_t p(e_t) < \beta \overline{R}$$

and so $c_t < (\beta\overline{R})^{t-1} c_1 < (\beta\overline{R})^{t-1} g(\varepsilon)\overline{w}$. Thus the present discounted value of consumption between date 0 and $T-1$ - call it V - is bounded above:

$$\begin{aligned} V = \sum_{t=0}^{T-1} \beta^t \ln c_t = \ln c_0 + \sum_{t=1}^{T-1} \beta^t \ln c_t < \ln \overline{w} + \sum_{t=1}^{T-1} \beta^t \ln[(\beta\overline{R})^{t-1} g(\varepsilon)\overline{w}] + \beta^T \ln \overline{w} \\ < \frac{1-\beta^T}{1-\beta} \ln \overline{w} + \ln(\beta\overline{R}) \sum_{t=1}^{T-1} (t-1)\beta^t + \beta \frac{1-\beta^{T-1}}{1-\beta} \ln g(\varepsilon) \end{aligned}$$

Suppose instead that the individual deviates and chooses $k_t = 0$ for $0 \leq t \leq T$, while keeping saving the same in subsequent periods. Call consumption under this alternative strategy $\hat{c}_t$: then we have $\hat{c}_t = c_t$ for $t \geq T$. The present discounted value of consumption between date 0 and $T-1$ under this alternative strategy - call it W - is bounded below: $W = \sum_{t=0}^{T-1} \beta^t \ln w_t \geq \dfrac{1-\beta^T}{1-\beta} \ln \underline{w}$. Thus the gain from the deviation is bounded below:

$$W - V > \frac{1-\beta^T}{1-\beta} \ln(\underline{w}/\overline{w}) - \ln(\beta\overline{R}) \sum_{t=1}^{T-1} (t-1)\beta^t - \beta \frac{1-\beta^{T-1}}{1-\beta} \ln g(\varepsilon)$$

As $\varepsilon \to 0$, $\ln(\underline{w}/\overline{w}) \to 0$, $\ln(\beta\overline{R}) \to 0$, $-\ln g(\varepsilon) \to -\ln g(0) > 0$. Thus for sufficiently low $\varepsilon > 0$, the gain from this deviation is positive, which contradicts the assumption that the original strategy was optimal. It follows that if $k_0 = 0$ and the conditions of the Lemma are satisfied, $k_1 = 0$. By induction, if $k_0 = 0$ we have $k_t = 0$ for all t. □

Corollary 6 (Dynamic sorting)**.** *Suppose $\theta_t \to \theta^*$. Then there exists T such that for $t > T$, if $k_t^h > k_t^l$, then $k_s^h \geq k_s^l$ for all $s > t$ with strict inequality unless $k_s^h = k_s^l = 0$.*

Proof. Define ε as in Lemma 12; there exists T such that for $t > T$, the conditions of Lemma 12 are satisfied, and individuals never have positive capital after they have zero capital. Let $k_t^h > k_t^l$. If at most one of h or l never has zero capital, then $k_s^h > k_s^l$ forever by induction on the static sorting Lemma. If $k_s^h = k_s^l = 0$ at some date s, then they both have zero capital for all dates $s' > s$. □

Definition 4. *$\mathcal{B}$ is the set of individuals with bounded capital. That is, for any $i \in \mathcal{B}$, there exists $\overline{k}^i$ such that $k_t^i < \overline{k}_t^i$ for all t. $\mathcal{U} = [0,1] - \mathcal{B}$ is the set of individuals with unbounded capital.*

Lemma 13 (Almost everyone bounded)**.** *Suppose $K_t \to K^*$ and $L_t \to 1$. Then $\mathcal{U}$ has measure zero. If it is nonempty, it includes the individual $i = 1$.*

Proof. Again, let ε be defined as in the previous Lemma; there exists T such that for $t > T$, the conditions of that Lemma are satisfied.

Suppose by contradiction that there exists a set $\mathcal{U}$ with positive measure - say m -whose capital is unbounded. Clearly we must have $k^i_t > 0$ for all $t \geq T$ and any $i \in \mathcal{U}$, otherwise we would have $k^i_s = 0$ for all $s > t$, and $1 + \max_{0 \leq t \leq s} k^i_t$ would be a bound.

Choose $i \in \mathcal{U}$ such that $\int \mathbb{I}\{k^j_T \geq k^i_T\}dj = m/2$. Since i's capital is unbounded, then for any $\bar{k}^i > 0$, there exists some date s at which $k^i_s > \bar{k}^i$. By the Dynamic Sorting Corollary, all individuals with $k^j_T > k^i_T$ also have $k^j_T > \bar{k}^i$. Thus at date s, aggregate raw capital is at least $\frac{m}{2}\bar{k}^i$, and aggregate effective capital is at least $\frac{m}{2}p(0)\bar{k}^i$. Since $\bar{k}^i$ was arbitrary, for any $\bar{K} > 0$, we can find a date s such that aggregate effective capital is higher than $\bar{K}$. This means K_t is unbounded, contradicting the assumption that $\theta_t \to \theta^* < \infty$. Thus $\mathcal{U}$ must have measure zero.

It remains to show that $1 \in \mathcal{U}$. First we argue that $k^1_t > 0$ for all t. If by contradiction there existed some first date t such that $k^1_t = 0$, then for any $i < 1$, we would have $k^i_{t-1} < k^1_{t-1}$. By one period sorting, this implies $k^i_t = 0$ for all $i < 1$, so aggregate capital must be zero at this date. If aggregate capital is zero at any date it is clearly zero forever (from the aggregate resource constraint). This implies that θ_t is zero forever (since all individuals have no capital, and will work full time, so $L_t = 1$), contradicting the assumption that $\theta_t \to \theta^* > 0$. Next, note that if $k^1_t > 0$ for all t, one period sorting again implies that since $k^1_0 > k^i_0$ for all $i < 1$, $k^1_t > k^i_t$ for all t. Thus if any individual is unbounded, $i = 1$ must also be unbounded. □

Lemma 14. *Suppose $\theta_t \to \theta^*$. Then all individuals with bounded capital eventually have effective interest rates bounded below β^{-1}. That is, for every $i \in \mathcal{B}$, there exists $T(i)$ and $\bar{R}^i < \beta^{-1}$ such that for $t > T(i)$, $p(e^i_t)R_t < \bar{R}^i$.*

Proof. If $i \in \mathcal{B}$, we have

$$p(e^i_t)R_t = p(e(R_t, w_t, k^i_t))R_t < p(e(R_t, w_t, \bar{k}^i))R_t \to p(e(\beta^{-1}, w^*, \bar{k}^i))\beta^{-1} < \beta^{-1}$$

where the last inequality comes because $\bar{k}^i < \infty$ implies $p(e_t) < 1$. Thus there exists $T(i)$ such that $t > T(i)$ implies

$$p(e^i_t)R_t < \bar{R}^i := \frac{1 + p(e(\beta^{-1}, w^*, \bar{k}^i))}{2}\beta^{-1} < \beta^{-1}$$

□

Lemma 15. *Suppose $\theta_t \to \theta^*$. Then all individuals with bounded capital eventually go to zero capital. That is, for all $i \in \mathcal{B}$, there exists $t(i)$ such that for all $t > t(i)$, $k^i_t = 0$ and $c^i_t = w_t$.*

Proof. Define $T(i), \bar{R}(i)$ as above. Suppose by contradiction that k_t^i never converges to zero after date $T(i)$. Then the unconstrained Euler equation holds in each period, and for any $s > 0$,

$$c_{T(i)+s}^i = c_{T(i)}^i \prod_{n=1}^{s} (\beta p(e_{T(i)+n}^i) R(T(i)+n) \leq c_{T(i)}^i \left(\beta \bar{R}(i)\right)^s$$

Since $\beta \bar{R}(i) < 1$ and $c_{T(i)}^i$ is finite, there exists s large enough that the right hand side is less than $w^*/2$, say, and thus the left hand side $c_{T(i)+s}^i$ is less than $w^*/2$. Since $w_t \to w^*$, we can find $t(i)$ such that for $t > t(i)$, $w_t > w^*/2$ and $c_t^i < w^*/2$. This is not optimal, since the individual can always deviate and consume $c_t = w_t$ in every period. This contradicts the assumption that k_t^i never reaches zero; so it must reach zero. Once it reaches zero, we know it stays there forever. □

Lemma 16. *Suppose $\theta_t \to \theta^*$. Then $\mathcal{U}$ is nonempty.*

Proof. An immediate corollary of the previous Lemma is that $L_t \to 1$, so $\theta_t \to \theta^*$ implies $K_t \to \theta^*$. If $\mathcal{U}$ is empty, k_t^1 permanently reaches zero at some point. By the Dynamic Sorting Corollary, all other individuals must reach zero at that point. Thus aggregate capital goes to zero, which contradicts the assumption that $K_t \to K_t^* > 0$. □

From Lemma 13, it follows that $i \in \mathcal{U}$, i.e. $k_t^1 \to \infty$. So we are done. □

H Proof of Lemma 5

First we show that any steady state with finance in which $e^i = 1$ for some i is a steady state of the $\tilde{p}$ economy. We claim that $b^i = a^i = 0 \forall i$. Suppose not: then $a^i > 0$ and $b^j > 0$ for some i, j. The Euler equations for i and j state that $\psi \beta^{-1} \frac{c_{t+1}^i}{c_t^i} = \psi R^A = R^B = \beta^{-1} \frac{c_{t+1}^j}{c_t^j}$, implying that consumption cannot be constant for both i and j, contradicting the definition of steady state. A similar argument implies that for each i, either $k^i > 0, e^i = 1$ or $k^i = e^i = 0$, as in the $\tilde{p}$ economy. It only remains to show that no individuals have $k^i \in (0, \underline{k}_{\tilde{p}})$ where $\underline{k}_{\tilde{p}}$ is the minimum positive level of capital in the $\tilde{p}$ economy. Suppose by contradiction that some individual i has such a level of capital. In the $\tilde{p}$ economy, that individual would have a profitable deviation which involves exerting effort $e_t^i < 1$ in some periods and decumulating capital, rather than exerting $e_t^i = 1$ in every period and maintaining a constant capital stock. In the ψ economy, the return to exerting $e_t^i = 1$ in every period and maintaining a constant capital stock is the same as in the $\tilde{p}$ economy, but the return to decumulating capital is weakly higher. This is because the return to effort $e < 1$ is $\tilde{p} = \max\{\psi^{-1}, p(e)\}$ in the $\tilde{p}$ economy, but (effectively) $\max\{R^A/R, p(e)\}$ in the ψ economy where $R^A/R \geq \psi^{-1}$. Thus, if we have a steady state of the ψ economy, all full-time managers certainly have enough wealth for this to be a steady state of the $\tilde{p}$ economy.

It is immediate that any steady state of the $\tilde{p}$ economy can be supported as a steady state of the ψ economy. Set $R^B = R$, $R^A = R/\psi$.

I Planning problem

The first order conditions of the planner's problem described in Section 6 are:

$$\frac{L}{c_t^L} = \lambda_t^L$$
$$\frac{1-L}{c_t^H} = \lambda_t^H$$
$$\lambda_t^L(1-L) - \widetilde{\psi}\lambda_t^H L \leq 0, T_t \geq 0, \text{ at least one equality}$$
$$\lambda_t^H = \beta \left\{ \lambda_{t+1}^L F_{LK}(k_{t+1}(1-L), L)(1-L) + \lambda_{t+1}^H F_{KK}(k_{t+1}(1-L), L)(1-L)k_t + \lambda_{t+1}^H F_K(k_{t+1}(1-L), L) \right\}$$

Combining the first three conditions yields $c_t^H \leq \widetilde{\psi} c_t^L$, as claimed in the main text. Since F is constant returns to scale, $F_{KK}(k_t(1-L), L)k_t(1-L) + F_{LK}(k_t(1-L), L)L = 0$. Using this fact, and substituting the remaining conditions into the fourth (assuming $T_t > 0$) yields

$$1 = \beta \frac{c_t^H}{c_{t+1}^H} \left\{ (\widetilde{\psi} - 1)F_{LK}(k_{t+1}(1-L), L)L + F_K(k_{t+1}(1-L), L) \right\}$$

Intuitively, the *social* marginal product of capital perceived by the planner is higher than the private marginal return. Because wages are received by workers, who have a higher marginal utility than managers who receive capital income (given the constraints on transfers), the planner would prefer to slightly increase capital, redistributing from capital to labor income. It follows that the capital-labor ratio in the optimal steady state is independent of L (noting that $F_{LK}(K, L)L = F_{LK}(K/L, 1)$) and satisfies $1 = \beta \left\{ (\widetilde{\psi} - 1)F_{LK}(\theta, 1) + F_K(\theta, 1) \right\}$. Welfare in one of these steady states is

$$\frac{\ln \left[(\widetilde{\psi} - 1)F_L(\theta, 1) + F(\theta, 1) - \theta \right]}{1-\beta} + \frac{\ln L - L \ln \widetilde{\psi}}{1-\beta}$$

Provided that $\ln \widetilde{\psi} < 1$ (a fairly weak assumption), the last term is increasing in L for $L < 1$; thus the optimal steady state has $L \to 1$. That is, the planner would like a vanishingly small mass of individuals to manage the capital stock on society's behalf and transfer almost all their capital income to the workers, who constitute almost all of the population.