Capital Controls and Free-Trade Agreements*

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Abstract

How does the conduct of optimal cross-border financial policy change with prevailing trade agreements? We study the joint optimal determination of trade tariffs and capital controls in a two-country, two-good model with trade in both goods and assets. Although the cooperative efficient allocation involves no intervention, a country planner acting unilaterally can achieve weakly higher domestic welfare by departing from free trade, in addition to levying capital controls. However, time-varying tariffs induce households to under or over-borrow through their effects on the real exchange rate. Specifically, in response to fluctuations where incentives to manipulate the terms of trade inter- and intra-temporally are aligned—such as when the availability of domestic goods changes, or when faced with trade disruptions to imports—optimal capital controls are larger when optimal tariffs are in place. In contrast, when the incentives are misaligned, the optimal trade tariff can partly substitute for the use of capital controls. These interactions also apply to small-open economies and to a Nash equilibrium in which countries engage in both capital-control and trade wars. Moreover, we show that the global welfare costs of capital-control wars are disproportionately larger when also accompanied by trade wars.

Key Words: Capital Controls; Free Trade; Ramsey Policy; Tariffs.
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1 Introduction

Trade and capital-flow management have long been key topics of macroeconomic policy and have, once more, come into sharp focus following the global financial crisis (GFC) and the Covid-19 pandemic. Following at least two decades of integration (Baier and Bergstrand, 2007), the process of trade liberalisation appears to have stalled. Events like the US-China trade war and recent supply-chain pressures[1] have contributed to substantially heightened uncertainty around world trade (Ahir, Bloom, and Furceri, 2018) and there has been a decline in the number of new regional trade agreements and a deceleration of global value chain integration[2] since the mid-2010s. Financial liberalisation too has abated in recent years, and the International Monetary Fund has partially revised their 'institutional view' to emphasise a role for taxing capital flows in specific circumstances (Qureshi, Ostry, Ghosh, and Chamon, 2011). This has been accompanied by an increase in the use of such measures in practice, especially those of a 'macroprudential' nature targeting cross-border flows (Ahnert, Forbes, Friedrich, and Reinhardt, 2020).

However, despite academic and policy debates around trade tariffs and capital controls growing in prominence, they have done so for largely independent reasons. Trade policy discussions often balance economic forces (e.g. comparative advantage) with political factors (e.g. consequences of de-industrialisation, trade sanctions), while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows.

In this paper, we show that adjustments to trade policy without changes in financial policy come at significant costs to efficiency and welfare. We provide a unifying framework to study the joint optimal determination of trade policy and capital controls, in a model where both instruments are driven by a common motive: to exploit a country's monopoly power in markets.[3] Within this setup, we assess how prevailing trade arrangements influence the incentives for, and the size of, optimal capital controls—accounting for, *inter alia*, economic size, strategic interactions and the implications for global welfare.

The starting point for our analysis is a canonical two-country, two-good endowment economy model, absent nominal or financial frictions. Households make an inter-temporal consumption-savings decision and choose their optimal consumption bundle intra-temporally. In the laissez-faire or decentralised allocation, relative consumption growth across countries is proportional to the relative decrease in price levels—i.e. the rate of real exchange rate depreciation. This condition, highlighted in Backus and Smith (1993) and Kollmann (1995), reflects the perfect risk-sharing underlying open-economy macroeconomic models with complete international asset markets. However, at this decentralised allocation, households do not internalise the effect of their actions on relative prices. These pecuniary externalities, described in (Geanakoplos and Polemarchakis, 1986), imply that a country planner maximising domestic welfare has an

[1] See Amiti, Redding, and Weinstein (2019) and Itskhoki and Mukhin (2022) for studies into the US-China trade war and recent sanctions on Russian goods, respectively.

[2] See D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek (2021) for further discussion.

[3] Our objective is not to argue that the manipulation of relative prices is the principle motive for policy intervention. Instead, we want to illustrate the fundamental interdependence of capital-flow taxes and tariffs through their influence on international relative prices, which will persist in more general environments. Moreover, this relationship persist even if tariffs are not optimally chosen.

incentive to manipulate the inter- and intra-temporal terms of trade, i.e. world interest rates and relative goods prices, respectively, even though the laissez-faire allocation is optimal from a global perspective.

Within this setup, Costinot, Lorenzoni, and Werning (2014) show that, when domestic households borrow between two periods, the planner tends to levy capital inflow taxes to delay consumption relative to the decentralised allocation, but must trade off the incentive to drive down the world interest rate with second-best effects on relative goods prices.[4] Specifically, capital controls introduce a wedge to the risk-sharing condition, so consumption growth can be slower than exchange rate depreciation.[5] Importantly, Costinot et al. (2014) focus on equilibria where a Free-Trade Agreement (FTA) is in place, ruling out goods-specific taxes. In this paper, we ask: what more can a planner achieve by deviating from a FTA, and what might this imply for the conduct of capital controls? Moreover, how do optimal tariffs evolve in a dynamic environment with capital controls?

Our key contribution is to relax the constraint imposed on the planner by a FTA, and assess the interactions between optimal capital controls and optimal trade tariffs within a tractable and transparent environment, using the primal approach of Lucas and Stokey (1983). To illustrate the mechanisms at play, we first assess the incentives of a country-planner acting unilaterally to maximise domestic welfare, without retaliation from abroad, before analysing implications for global welfare in a strategic setting. Within the unilateral setting, the country planner can achieve weakly higher welfare if they implement goods-specific taxation—precluded under a FTA—in conjunction with capital controls.

Building on this, we describe the interaction between financial and trade policy at the planning allocation. Consider again a scenario in which domestic households borrow between two periods, this time specifically because the good consumed with home bias (the 'domestic good') is temporarily low. In this situation, a planner acting unilaterally will seek not only to delay aggregate consumption by taxing capital inflows, but also to delay consumption of the relatively expensive domestic good. Absent a FTA, the planner achieves this by levying a temporary subsidy on imports. Because the domestic good is consumed in greater proportion by domestic consumers, this intervention will put pressure on the real exchange rate to depreciate. Recall the risk-sharing condition relating relative consumption growth, the rate of depreciation and a wedge reflecting capital flow taxes. If the planning allocation for consumption without a FTA is not too different from that with a FTA, trade policy will force an adjustment in the level of capital controls due to its effects on the path for real exchange rates.[6] Intuitively, the exchange rate depreciation encourages over-borrowing by households whose consumption

[4]Costinot et al. (2014) note that optimal capital controls are not guided by the absolute desire to alter the inter-temporal price of goods produced in a given period, but rather by the relative strength of this desire between two consecutive periods, generalising the results from a two-period environment in Obstfeld and Rogoff (1996).

[5]This wedge can also be understood as a measure of exchange-rate misalignment induced by the planner at the optimal allocation (e.g. Corsetti, Dedola, and Leduc, 2020).

[6]This insight applies to the interaction of any policy interventions which does not directly induce a wedge in the risk-sharing condition (e.g. monetary policy, fiscal policy) not just trade policy.

bundle becomes cheaper relative to the future.[7]

Whether capital controls are larger or smaller when the FTA is relaxed depends on the alignment of the unilateral planner's incentives to manipulate the terms of trade inter- and intra-temporally. In the above example in which capital flow taxes are larger in the presence of tariffs, incentives are aligned. Inter-temporally, the planner leans against the private desire to over-borrow today while, intra-temporally, the planner offsets the private desire to consume the relatively expensive domestic good. In contrast, when the domestic endowment of the second 'foreign good' is temporarily low, inter- and intra-incentives misaligned. In this case, optimal tariffs generate real exchange rate movements that would—absent further action—incentivise under-borrowing. So, at the optimal allocation, trade policy acts as a partial substitute for capital controls, requiring smaller capital flow taxes than the FTA case. Calibrating our stylised framework to standard values used in the literature, we show these interactions can be quantitatively important, with the size of capital flow taxes across the two regimes differing by over 25%.

The same logic applies when there are time-varying trade disruptions, such as multilateral sanctions or global supply-chain disruptions, where we show that both capital controls and tariffs play a role in the optimal policy response of the unilateral planner. Consider the case of an exogenous and temporary increase in the cost of imports. Here, because aggregate consumption is relatively expensive in the near term, the planner will tax capital inflows to delay consumption into the future, as long as the inter-temporal elasticity of substitution is sufficiently low. However, the planner also has the intra-temporal incentive to reduce the price of domestic-good consumption, in order to offset the higher cost of imports, and achieves this via an import subsidy. As a consequence of the alignment of inter- and intra-temporal incentives, our theory therefore suggests that capital flow taxes will be *larger* absent an FTA in response to trade disruptions.

Surprisingly, we also show that our results for the interaction between financial and trade policy can still apply in small-open economies, when the unilateral planner's ability to manipulate inter-temporal margins disappears. At the knife-edge case of unitary trade and inter-temporal elasticities of substitution (Cole and Obstfeld, 1991), we show that when inter- and intra-temporal planning incentives are aligned, the optimal capital inflow tax is invariant to the size of the economy. This is because the capital flow tax needed to address the inter-temporal margin exactly coincides with that required to address intra-temporal incentives. While the optimal tariff does change absent a FTA, it is still non-zero since countries are always large in their domestic goods market. As a result, our conclusions about the interaction between policy instruments qualitatively apply to this case as well. However, in cases where inter- and intra-temporal incentives are misaligned, the optimal capital inflow tax falls as the country becomes small.

To ascertain how these incentives play out in the global economy, we analyse a Nash equi-

[7] We use the term 'over-borrowing' because a larger capital flow tax would be required to induce a given path for consumption, given the path of real exchange rates. The literature on over-borrowing in open economies is most recently surveyed most recently in Rebucci and Ma (2019) and Bianchi and Lorenzoni (2021).

librium where both countries' planners set policy as a best response to the other. While the underlying mechanisms from the unilateral equilibrium persist in the strategic setting, we find that capital flow taxes tend to be larger absent a FTA in all states of the economy (when both the domestic or foreign goods are below their long-run level) due to the effects of tariff competition on the real exchange rate. Moreover, country competition with capital controls and tariffs follows an 'inverse elasticity rule'. All else equal, capital controls are larger when the elasticity of inter-temporal substitution is low and tariffs are more prevalent when the intra-temporal elasticity of substitution between goods is low.[8]

Finally, we analyse the implications for global welfare. We show that, while the joint application of capital controls and tariffs may be unilaterally optimal for an individual country when there is no retaliation, the costs to global welfare are disproportionately large. Trade and financial policy are not simply redistributive: the domestic welfare gains from intervention are always small in comparison to foreign losses. Moreover, we show in the strategic setting that concurrent capital control and trade wars result in disproportionately larger welfare losses, for each country, than capital control wars alone. In contrast, the cooperative optimal allocation involves no capital flow taxes or trade tariffs, implying that the pecuniary externalities underlying our analysis are a feature of well-functioning international markets.

Related Literature. Our work is most closely related to Costinot et al. (2014) who study the role of capital controls as dynamic terms-of-trade manipulation in large-open endowment economies.[9] While policy in our setup is driven by the same pecuniary externalities, we depart from the assumption of free trade and study an environment with goods-specific taxes.

A key feature of our work is to combine analyses of inter-temporal incentives to manipulate the terms of trade, which is a key part of the broader literature on capital controls surveyed in Rebucci and Ma (2019) and Bianchi and Lorenzoni (2021),[10] with intra-temporal incentives, for which tariffs are regularly applied in practice (see Broda, Limao, and Weinstein, 2008). By doing this, a novelty of our analysis is the study of the interaction between policy instruments. In this sense, our paper is also closely related to Jeanne (2012), who shows that trade tariffs can achieve the same real exchange rate devaluation as balance-sheet policies, but in a one-good world and without a discussion of optimal policy implementation.

Unlike our paper, the literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (and often heterogeneous) firms. Demidova and Rodriguez-Clare (2009) show that optimal tariffs trade-off

[8] The findings mirror those in the optimal taxation literature (e.g. Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), where the planner taxes inelastic commodities more.

[9] Heathcote and Perri (2016) study capital controls in a two-country, two-good model with incomplete markets and capital. But, unlike our paper and Costinot et al. (2014), they do not derive the optimal policy.

[10] Mendoza (2002) and Bianchi (2011) study small-open economies where goods prices appear in borrowing constraints. These models highlight how incentives to manipulate the terms of trade via capital controls can have first-order effects on countries' ability to borrow. Farhi and Werning (2014), Farhi and Werning (2016) and Schmitt-Grohé and Uribe (2016), amongst others, study the use of capital controls to correct aggregate-demand externalities in models with nominal rigidities. Marin (2022) discusses how capital controls can be used in the US in addition to monetary policy in the face of dollar scarcity.

a domestic mark-up distortion and incentives to increase the number of imported good varieties. Introducing roundabout production, Caliendo, Feenstra, Romalis, and Taylor (2021) show that the optimal tariff is smaller and can even be negative. Our results, with trade in assets, can be interpreted in a similar vein: relative to the case of financial autarky, tariffs become second-best instruments due to their effects on the cost of borrowing. However, our results highlight that the optimal tariff can be either smaller or larger than the financial autarky counterpart, depending on the state of the economy.

Finally, our paper contributes to a growing literature assessing the joint role of trade and macroeconomic stabilisation policies. Bergin and Corsetti (2020) study the optimal response of monetary policy to tariff shocks and find that the optimal response to a unilateral tariff imposed by a trade partner is to engineer a depreciation to offset its effects. Auray, Devereux, and Eyquem (2020) study the scope for trade wars, modelled via optimal strategic tariffs, and currency wars in a New-Keynesian small-open economy model using a first-order approximation. However, their model features balanced trade, so there is no scope for capital control wars, unlike in our paper. Also focusing on a first-order approximation for small-open economies, Jeanne (2021) studies monetary policy and the accumulation of foreign reserves, but emphasises the distinction between a 'Keynesian regime' where instruments are used to achieve full employment and a 'classical regime' where tariffs are used to manipulate the terms of trade. Unlike these papers, we pay particular attention to interactions between policy instruments at the optimal allocation, without resorting to approximation methods.

Outline. The remainder of the paper is structured as follows. Section 2 describes the two-country, two-good environment. Section 3 characterises the optimal unilateral planning allocation. Section 4 discusses policy implementation at the optimal allocation. Section 5 studies strategic cross-country interactions, discussing the scope for capital control and trade wars. Section 6 considers global welfare and cross-border spillovers. Section 7 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. Time is discrete and infinite, $t = 0, 1, ...$, and there is no uncertainty. The preferences of the representative Home consumer are denoted by the time-separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t is aggregate Home consumption and $u(C)$ is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C \to 0} u'(C) = \infty$. $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with asterisks denoting Foreign variables.

Consumers in both countries consume two goods, good 1 and good 2. We denote the representative Home consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them into the vector $\mathbf{c}_t = [c_{1,t}\ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t}\ g_{2,t}]'$, where $g_{i,t} = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for $i = 1, 2$, while second derivatives are written as $g_{ij,t} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t} \partial c_{j,t}}$ for $i, j = 1, 2$. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogous derivatives.

We consider an environment where both countries can be endowed with both goods.[11] Throughout, without loss of generality, we assume that Home consumers have a 'home bias' for good 1, and we describe this as the 'domestic good'. Defining the Home expenditure share on domestic goods as α, then 'home bias' implies $\alpha > 0.5$. Likewise, Foreign consumers prefer good 2 (the 'foreign good'). We assume these preferences are symmetric across countries such that the Foreign expenditure share on foreign goods is $\alpha^* = \alpha$. The Home (Foreign) consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ ($y_{1,t}^*$) and $y_{2,t}$ ($y_{2,t}^*$), respectively, and are weakly positive in all periods. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively.

We assume that both countries begin with zero assets in period 0, with the budget constraint for the Home household expressed as:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \leq 0 \tag{1}$$

where $\mathbf{p}_t = [p_{1,t}\ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]'$ is the vector of Home endowments. The Foreign budget constraint is analogous.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and, since good 1 is the 'domestic good' and good 2 the 'foreign good', we refer to an increase in S_t as a deterioration of the Home terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} :\ g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

Free-Trade Agreements and the Pareto Frontier. A key novelty of this paper is to study how prevailing trade agreements influence the incentives of a planner levying taxes on capital flows. In the presence of a FTA, households' consumption allocations are Pareto efficient (from an individual-household perspective) and can be summarised by:

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \geq C_t\} \tag{2}$$

[11] We focus on an endowment setup for mathematical tractability. However, as discussed in Section 4.6, our findings carry over to frictionless production environments, e.g. with linear production technology in productivity and labour. Moreover, the case of perfect specialisation is the limit where the endowment of one good goes to zero.

for some C_t, where $\mathbf{Y}_t = [Y_{1,t}\ Y_{2,t}]'$. This problem yields a Pareto Frontier, which summarises efficient combinations of consumption $\{c_{1,t}, c_{2,t}\}$ for a given level of aggregate consumption C_t, which coincides with the contract curve for the representative Home and Foreign consumers when there are no goods-specific taxes. The full expressions for the Home and Foreign Pareto Frontiers are summarised by $\mathbf{c}(C)$ and $\mathbf{c}^*(C^*)$ in Appendix A.1, which reflect individual households' optimisation of consumption bundles given an aggregate consumption C.

3 Optimal Allocations with a Unilateral Planner

We begin by considering an equilibrium in which the Home planner seeks to maximise domestic welfare, while the Foreign planner is assumed to be passive—i.e. does not levy taxes in response to Home policy. Applying a primal approach to characterise the optimal policy, we compare the equilibrium with a FTA in place (corresponding to the two-good environment studied in Costinot et al., 2014) to an equilibrium where the Home planner is unconstrained by a FTA.

In both cases, the equilibrium conditions of the representative Foreign household act as a constraint for the unilateral Home planner. Foreign households undertake a standard optimisation, maximising Foreign discounted utility subject to their inter-temporal budget constraint at world prices $\mathbf{p}_t$. With λ^* denoting the Lagrange multiplier on the Foreign inter-temporal budget constraint, the resulting first-order conditions are:[12]

$$\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{3}$$

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0 \tag{4}$$

3.1 With Free Trade

In the presence of a FTA, the Home government chooses the sequence of Home aggregate consumption $\{C_t\}$ to maximise the discounted lifetime utility of the Home representative consumer subject to: (i) the representative Foreign consumer's utility maximisation at world prices; (ii) market clearing in each period; and (iii) the Pareto Frontier arising from the FTA. The Foreign optimality conditions, equations (3) and (4), the domestic inter-temporal budget constraint, equation (1), and the market-clearing conditions can be summarised in a single implementability condition (Lucas and Stokey, 1983), described in the following proposition:

Proposition 1 (Implementability for Unilateral Planner) *When the Foreign country is passive, an allocation $\{\mathbf{c}_t, \mathbf{c}_t^*\}$, together with world prices $\mathbf{p}_t$, form part of an equilibrium if they satisfy*

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

where $\boldsymbol{\rho}(C_t) \equiv u^{*\prime}(C^*(C_t))\nabla g^*(\mathbf{c}_t^*(C_t))$ denotes the price of consumption at each t.

[12] See Appendix A.2 for a full statement of the representative Foreign household's optimisation problem.

Proof: See Appendix A.2. □

With this definition, the Home planning problem can then be written as:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

where the third line (FTA) summarises the Pareto Frontier constraint imposed by the presence of a FTA. After substituting (FTA) into (IC), we assume that $\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}(C_t) - \mathbf{y}_t]$ is a strictly convex function of C_t to guarantee a unique solution to (P-Unil-FTA).

Optimal Allocation. Because utility is time-separable, the first-order condition is:

$$u'(C_t) = \mu \mathcal{MC}_t^{FTA} \tag{5}$$

where μ is the multiplier on the implementability constraint and:

$$\begin{aligned} \mathcal{MC}_t^{FTA} \equiv & u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*(C_t)) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*) C^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*(C_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*) \frac{\partial \nabla g^*(\mathbf{c}_t(C_t))}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t] \end{aligned}$$

The left-hand side of equation (5) is the marginal utility from one additional unit of aggregate consumption for the representative Home consumer. The right-hand side represents the marginal cost of that unit of consumption, captured by $\mathcal{MC}_t^{FTA}$. The first term in $\mathcal{MC}_t^{FTA}$ is the price of one unit of consumption. It can be shown to be equal to $u^{*\prime}(C_t^*)Q_t^{-1}$. The second term reflects how the inter-temporal price of consumption changes when importing one additional unit of consumption, for given relative goods prices. The final term reflects how relative goods prices change with aggregate consumption. If endowments and consumption outcomes coincide, $\mathbf{c}_t = \mathbf{y}_t$, equation (5) collapses to $u'(C_t) = \mu u^{*\prime}(C_t^*)Q_t^{-1}$, which corresponds to the decentralised allocation. Moreover, $\mu = 1$ coincides with perfect risk sharing.

3.2 Without Free Trade

Without free trade, the Home planner—unconstrained by the Pareto Frontier—can directly choose the allocation of both goods 1 and 2. The Home planner's problem is:

$$\max_{\{c_{1,t}, c_{2,t}\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$C_t = g(\mathbf{c}_t) \tag{nFTA}$$

where the third line (nFTA) reflects that aggregate consumption C_t can then be backed out of the consumption aggregator $g(\mathbf{c}_t)$. The implementability condition is unchanged and, as in the FTA-case, we assume that $\boldsymbol{\rho}(g(\mathbf{c}_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex to ensure a unique solution to the planning problem.

Optimal Allocation. The first-order conditions—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t)g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} \tag{6}$$

$$u'(C_t)g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} \tag{7}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:[13]

$$\begin{aligned}
\mathcal{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_1^*(\mathbf{c}_t) + u^{*\prime\prime} g_1^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
\mathcal{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t) + u^{*\prime\prime} g_2^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]
\end{aligned}$$

Like equation (5), equations (6) and (7) equate the marginal benefit from a unit of good-specific consumption to its marginal cost—for goods 1 and 2, respectively. Without free trade, the planner optimises over the consumption allocation good by good. Take equation (6), for example. As before, the first term on the right-hand reflects the price of one unit of good 1. The next term reflects how the inter-temporal component of that price (i.e. the cost of borrowing) changes. The final term, captures the intra-temporal margin—specifically how each good-specific price changes with respect to c_1.

3.3 Comparing Optimal Allocations

For the Home planner, facing no retaliation, the first-order condition under a FTA, equation (5), represents a constrained first-best allocation. However, the no-FTA optimality conditions, equations (6) and (7), represent the first-best outcome for the Home country, as the following proposition explains.

Proposition 2 (Optimal Capital Controls without a FTA) *In the absence of a FTA, the unilateral optimal allocation* $\mathbf{c}_t$ *satisfies* (6) *and* (7)*. Moreover:*

(i) *the level of welfare* U_0 *achieved in* (P-Unil-nFTA) *is always weakly higher than that achieved in* (P-Unil-FTA)*;*

[13] For notational ease, we do not make explicit the dependence of $\mathbf{c}_t^*$ on $\mathbf{c}_t$, which arises through market clearing.

(ii) *if the optimal allocation* $\mathbf{c}$ *in* (P-Unil-nFTA) *violates the Pareto frontier* (2) *given by a FTA, then* (i) *holds strictly; and*

(iii) *the welfare achieved, and corresponding allocation* $\mathbf{c}$*, in* (P-Unil-FTA) *and* (P-Unil-nFTA) *coincide when endowments are proportional to consumer preferences,* $y_1 \propto \alpha$, $y_2 \propto 1-\alpha$, $y_1^* \propto 1-\alpha$ *and* $y_2^* \propto \alpha$.

Proof: See Appendix A.3. □

To illustrate Proposition 2, Figure 1 plots the optimal allocations with (blue) and without (green) a FTA, alongside the loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (grey, and black for $C = 1$), in the long-run where endowments are constant. For this, and all subsequent numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{8}$$

where $\phi > 0$ is the elasticity of substitution between good 1 and 2.

In Figure 1, the blue line maps the Pareto Frontier: the efficient combinations of $\{c_1, c_2\}$ for different levels of long-run aggregate consumption C, which are consistent with a FTA. But when not constrained by a FTA, the planner can achieve a higher level of consumption by changing the Home allocation $\{c_1, c_2\}$, as in parts (i) and (ii) of Proposition 2. For $y_1 > \alpha$—the area above the black line, where good 1 is abundant—the long-run allocation absent FTA is more biased towards c_1. Whereas for $y_1 < \alpha$—the area below the black line, where good 1 is scarce—the allocation is more biased towards c_2. The FTA and no-FTA allocations only coincide in the case $y_1 = y_2^* = \alpha$—part (iii) of Proposition 2.

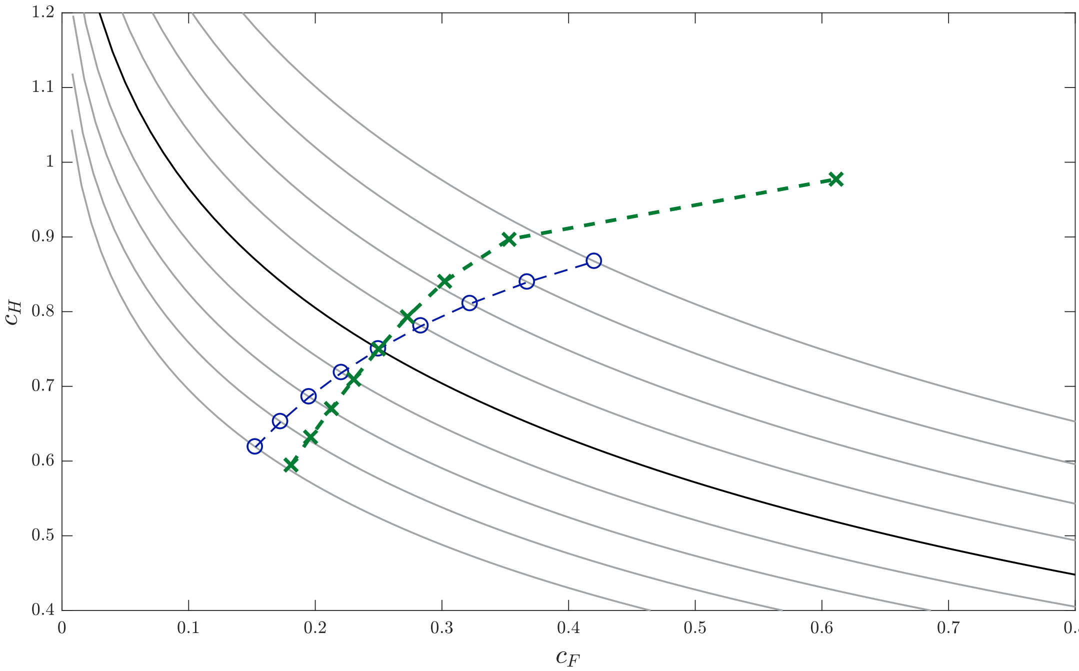
4 Policy and Macro Outcomes at the Optimal Allocation

In this section, we describe the implementation of the optimal allocation and highlight how the policy instruments interact with one another. We then contrast the macroeconomic dynamics at the planning allocation with and without a FTA, and we compare these to the decentralised allocation. We conclude the section by discussing the generality of our results.

4.1 Implementation

While it well-known that implementation of the Ramsey optimal allocation via taxation is generally non-unique (Chari and Kehoe, 1999), we consider a policy-relevant implementation where policy instruments have observable real-world analogues. We assume households can trade in non-contingent bonds, denominated in each good variety. The Home planner can

Figure 1: Optimal Allocations and the Pareto Frontier



Notes: Plot of optimal consumption allocations for Home consumer from Ramsey capital flow taxation (i) with a FTA in place (blue circles, i.e. the Pareto frontier) and (ii) absent a FTA, with goods-specific taxation (green crosses) at different Home endowments. Specifically using nine equally-spaced allocations for $y_1 \in [\alpha - 0.25, \alpha + 0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.75$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for $C = 1$, grey otherwise).

impose the same proportional tax θ_t on the gross returns to net lending in all bond markets. So the per-period budget constraint for the Home consumer can be written as:

$$\tilde{\mathbf{p}}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \tilde{\mathbf{p}}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\tilde{\mathbf{p}}_t \cdot \mathbf{a}_t) - T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ when a FTA is in place, $\mathbf{a}_t$ denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t\to\infty} \tilde{\mathbf{p}}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximisation are given by:

$$u'(C_t) g_i(\mathbf{c}_t) = \beta(1 - \theta_t)(1 + r_{i,t}) u'(C_{t+1}) g_i(\mathbf{c}_{t+1}) \tag{9}$$

for $i = 1, 2$, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation yields the Backus and Smith (1993) condition with a wedge reflecting capital-flow taxation:[14]

$$(1 - \theta_t) = \frac{u'(C_t)}{u'(C_{t+1})} \frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C_t)} \frac{Q_t}{Q_{t+1}} \tag{10}$$

[14]This follows from $g_i/p_i = 1/P$.

A tax on capital inflows (or a subsidy for outflows) is then captured by values of $\theta_t < 0$, which can also be interpreted as a tax on current consumption relative to future consumption.

Without a FTA, the Home planner can additionally levy a proportional import tax τ_t, and $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$ where $\boldsymbol{\tau}_t = [1 \quad \tau_t]'$, and an import tariff is captured by $\tau_t > 0$. The representative Home household faces an import price $p_{2,t}(1+\tau_t)$, so their relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1-\alpha}\left(\frac{1}{S_t(1+\tau_t)}\right)^{-\phi} \tag{11}$$

4.2 Decomposing the Risk-Sharing Wedge

To investigate the interactions between the capital flow tax and tariffs, we decompose the risk-sharing condition, equation (10) into two wedges. Taking logs:

$$\ln(1-\theta_t) \approx -\theta_t = \underbrace{-\sigma\left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}^*_{t+1} - \hat{C}^*_t\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}} \tag{12}$$

where $\hat{x}$ denotes the natural logarithm of x. The 'consumption wedge' component captures incentives to tax capital inflows pertaining to the evolution of target relative consumption over time. The 'RER wedge' reflects capital flow taxation incentives linked to the evolution of the real exchange rate Q.

Consider again the case where the fraction of good 1 owned by the Home country $y_{1,t}/Y_{1,t}$ is temporarily low—holding the overall stock of good 1 fixed over time ($Y_{1,t} = Y_1$ for all t). Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption. However, each additional unit of consumption brought forward raises the cost of borrowing, captured by (5). Additionally, the Home household will buy relatively more units of the the domestic good (good 1) from abroad, at a time when it is relatively more expensive to do so, captured by (6).[15] As a result, when the good-1 endowment deviates from its long-run level, the planner's inter- and intra-temporal incentives to manipulate the terms of trade are aligned, as the planner chooses to delay aggregate consumption *and* bid down the price of good 1. In the absence of a FTA, the planner additionally levies a temporary import subsidy, which depreciates the terms of trade by more than otherwise. Therefore, relative to the FTA-case, the RER wedge is higher. For a given target consumption path (i.e. if the consumption wedge does not change much across the FTA and no-FTA regimes), a larger capital-flow tax is required when tariffs are optimally set.

In contrast, suppose the fraction of the foreign good (good 2) owned by the Home country ($y_{2,t}/Y_2$) temporarily falls. While the planner's inter-temporal incentive to delay consumption is the same as before, the intra-temporal incentive differs since the Home country will now sell relatively more of good 1 abroad. The planner has an incentive to act monopolistically, drive up the price of good 1 and appreciate the terms of trade. If the consumption wedge is relatively unchanged, optimal capital controls must then be smaller absent a FTA—so tariffs

[15] Specifically, the fall in y_1 is greater than the fall in c_1 so that $(c_1 - y_1)$ increases.

act as a partial substitute for capital-flow taxation. Consistent with this reasoning, we show in subsequent simulations that the consumption wedge changes little across the FTA and no-FTA regimes, whereas the RER wedge differs substantially.

4.3 Model Simulation

To illustrate the macroeconomic dynamics and implementation of the optimal allocations, we first describe two general simulation scenarios that capture the key intuition. Our simulations are deterministic. We specify initial and terminal values for the country-good endowments, and construct the full sequence of endowments for all periods by assuming that endowments follow a first-order autoregressive process:

$$\begin{aligned} y_{i,t+1}^{(*)} &= \left(1-\rho_i^{(*)}\right)\overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1,2, \\ \mathbf{y}_0 &= [y_{1,0}\ y_{2,0}]' \\ \mathbf{y}_0^* &= [y_{1,0}^*\ y_{2,0}^*]' \end{aligned}$$

where for simplicity we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$. In both scenarios, we assume there is no change in the aggregate endowment ($Y_{1,t}$ and $Y_{2,t}$ are constant). As a result, with households able to fully insure their consumption against known changes in their endowment, perfect consumption smoothing is achieved in the decentralised allocation.[16]

Based on the CRRA per-period utility function and the Armington (1969) specification for aggregate consumption, the model calibration for both scenarios is detailed in Table 1. In each, we compare the decentralised allocation, the unilateral Ramsey planning allocation with a FTA in place, and one without a FTA. To compare the dynamic implications of the three variants in a consistent manner, we must also equalise the long-run equilibrium (i.e. 'steady state') of each model by using a steady state import tariff for the Home country.

Table 1: Benchmark Model Calibration

Parameter	**Value**	**Description**
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

4.4 Scenario 1: Temporarily Low Endowment of Domestic Good

Our first scenario simulates the Home economy recovering from a domestic downturn. Specifically, the Home country's endowment in good 1 is low in the near term, and grows towards

[16] This assumption merely serves to sharpen comparison with the decentralised allocation and clarify the mechanisms driving our key results. The same factors are at play when the aggregate endowment is allowed to fluctuate.

its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for $i = 1, 2$, we assume that $y_{1,0} = 0.75\overline{y}_1$ and $y_{2,0} = \overline{y}_2$. To ensure there is no aggregate uncertainty: $y_{1,0}^* = 1 - y_{1,0}$ and $y_{2,0}^* = 1 - y_{2,0}$. The resulting time profiles for the allocations are plotted in Figure 2.

The optimal policy, both with and without a FTA, involves leaning against capital flows to delay consumption into the future. This is demonstrated in the bottom-left panel, plotting the evolution of the balance of payments, which varies by less under the two planning solutions relative to the decentralised outcome. Additionally, because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an incentive to restrict the global excess demand for good 1 over and above their endowment y_1. Driving these incentives is the planner's expectation that the future price of c_1 and C will fall. Therefore, the planner taxes aggregate consumption C via a capital inflow tax $\theta < 0$ and, in the absence of a FTA, levies an increasing path for import tariffs.

In the presence of a FTA, the planner achieves the desired allocation by choosing a lower level of aggregate consumption C in the near term, which entails a disproportionately lower c_1 on account of Home consumers' preference for good 1. When unconstrained by a FTA, the planner can restrict the net global supply of good 1 via an import tariff, which incentivises Home consumers to purchase a larger fraction of the good-1 endowment on the global market. While the required capital control taxes are generally small—between 6 and 8% on impact for the planner with and without an FTA, respectively—the goods tax is large and variable in the absence of a FTA—over 50% in the long run and increasing from around 15%.

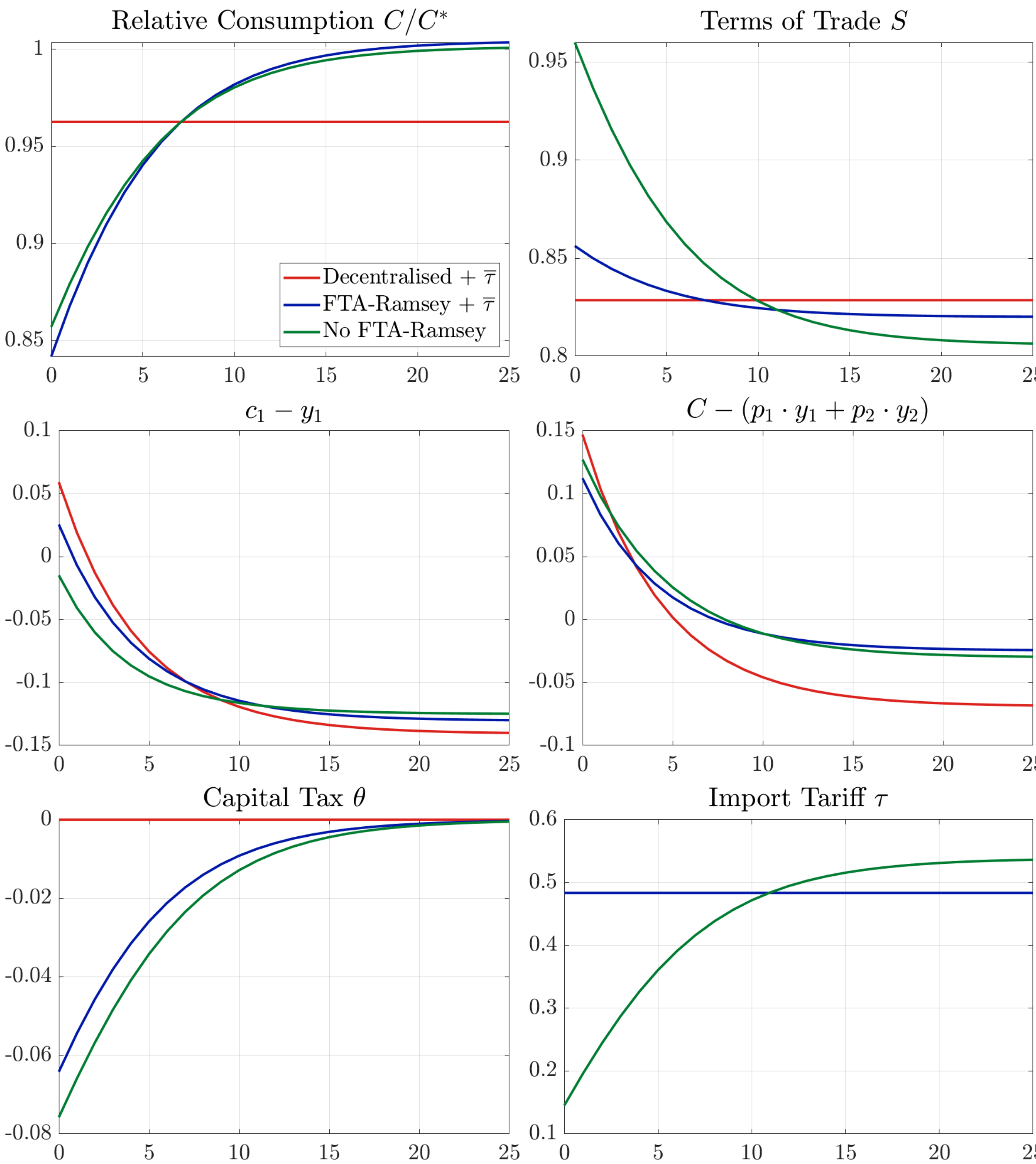
Figure 3 plots the two wedges from equation (12) from this scenario. The left-hand plot indicates that it is the consumption wedge that explains most of the variation in the capital flow tax θ, regardless of whether a FTA is in place or not. This arises because the differences between the optimal path of aggregate consumption in the FTA and no-FTA cases is small. However, the differences between the RER wedge in either case—shown in the right-hand panel—are more marked. In particular, it is primarily because the RER wedge is larger in magnitude when no FTA that, in this scenario, capital flow taxes at the optimal allocation are larger than with free trade. Nevertheless, both the consumption and RER wedge have the same sign, reflecting the alignment of incentives to manipulate the inter- and intra-temporal terms of trade.

4.5 Scenario 2: Temporarily Low Endowment of Foreign Good

Our second scenario simulates the case in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. This is akin to a positive Foreign export-sector shock, as the Foreign country's endowment of good 2 is high in the near term, but falls towards its long-run level. We assume that $y_{2,0}^* = 1.25\overline{y}_2^*$ and $y_{1,0}^* = \overline{y}_1^*$. To ensure there is no aggregate uncertainty: $y_{1,0} = 1 - y_{1,0}^*$ and $y_{2,0} = 1 - y_{2,0}^*$. The resulting time profiles for the allocations are plotted in Figure 4.

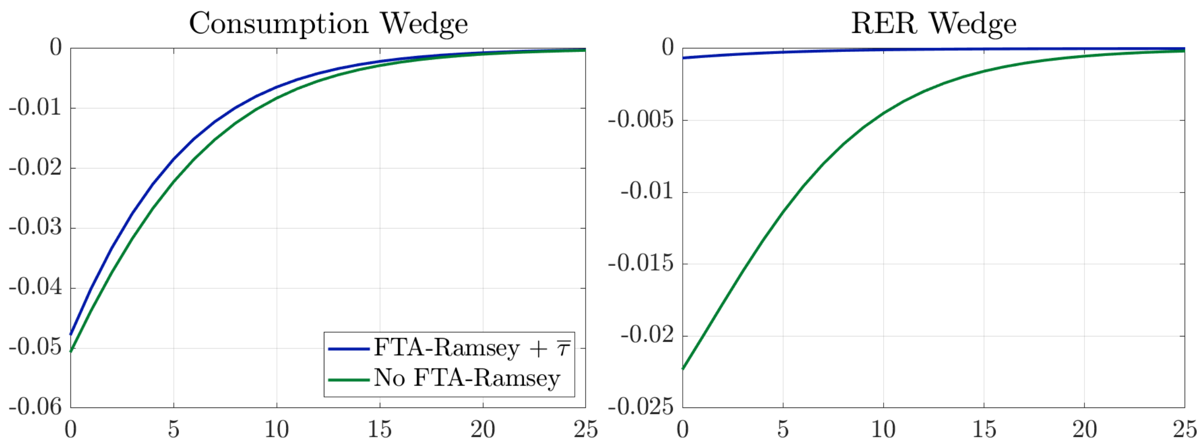
As in scenario 1, the Home country borrows in the near term in the decentralised allocation, knowing that their endowment will increase in the future. However, the net supply of good 1

Figure 2: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for macroeconomic outcomes in Scenario 1, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

Figure 3: Decomposition of Optimal Capital Flow Taxes for Scenario 1



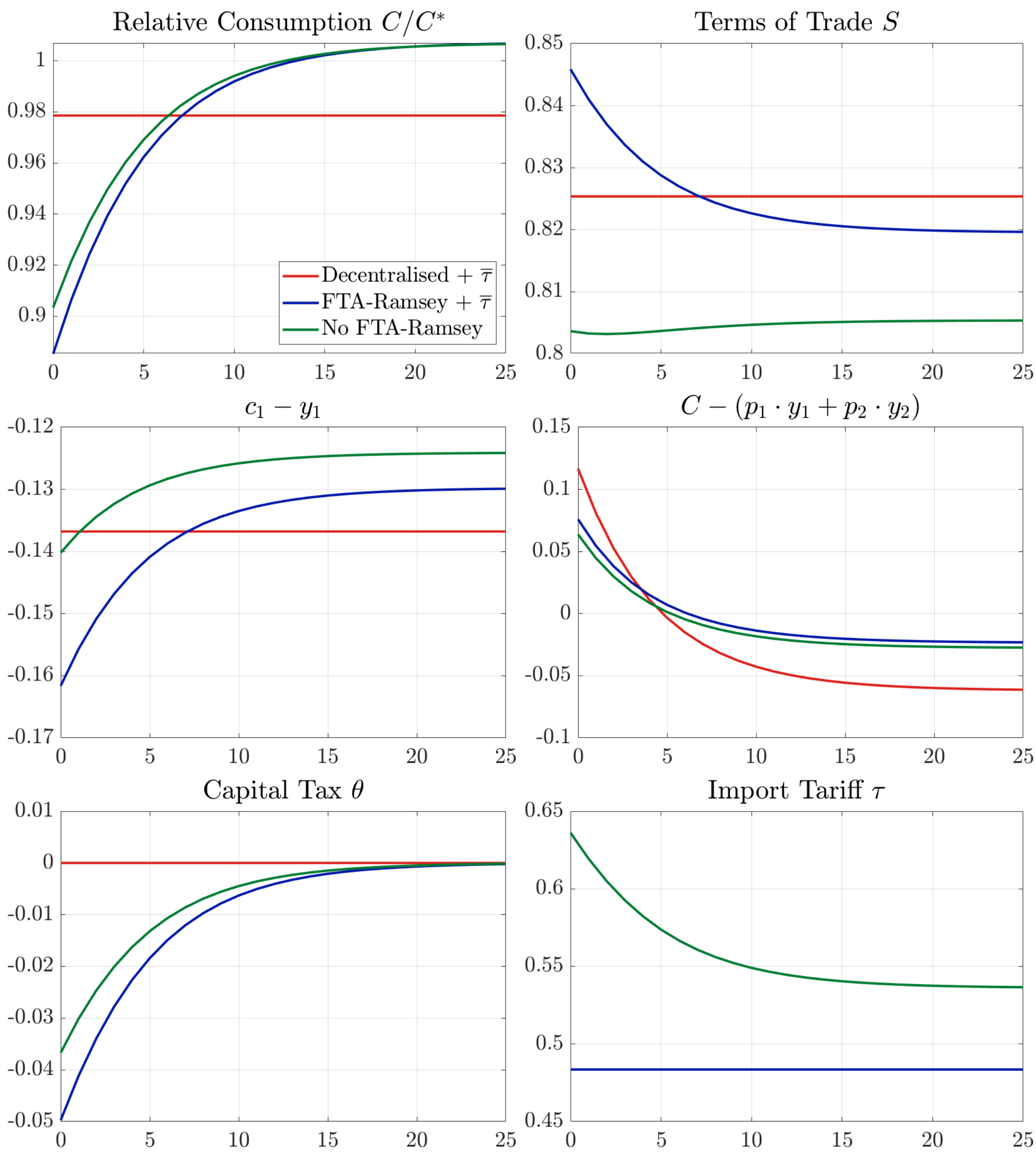
Notes: Time profile for Home capital flow tax components in Scenario 1, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

that Home sells abroad rises, because c_1 falls while y_1 is unchanged. The Home planner wants to delay aggregate consumption C inter-temporally, but also has an intra-temporal incentive to drive up the relative price of good 1. Absent a FTA, the planner levies a high import tariff in the near term to increase c_1 and drive up its relative price. But the optimal capital inflow tax is smaller absent a FTA, as it must strike a balance between restricting C and boosting c_1.

This interaction between instruments can be seen by inspecting the risk-sharing wedges defined in equation (12), which are shown in Figure 5. In this scenario, as in Scenario 1, the consumption wedge explains the majority of overall variation in the capital flow tax. And, again, because the differences in the optimal path of aggregate consumption between the FTA and no-FTA cases are small, the differences between the consumption wedges is small too. However, in contrast to Scenario 1, the right-hand panel of Figure 5 demonstrates that the RER wedge has the opposite sign for the planner when there is no FTA. This reflects the misalignment of inter- and intra-temporal incentives in this scenario. As a consequence of this, the planner can levy tariffs to stabilise the terms of trade intra-temporally and, at the same time, offset the need to use capital flow taxes to manipulate relative prices for inter-temporal incentives. This then, mechanically, results in the smaller capital outflow tax without a FTA.

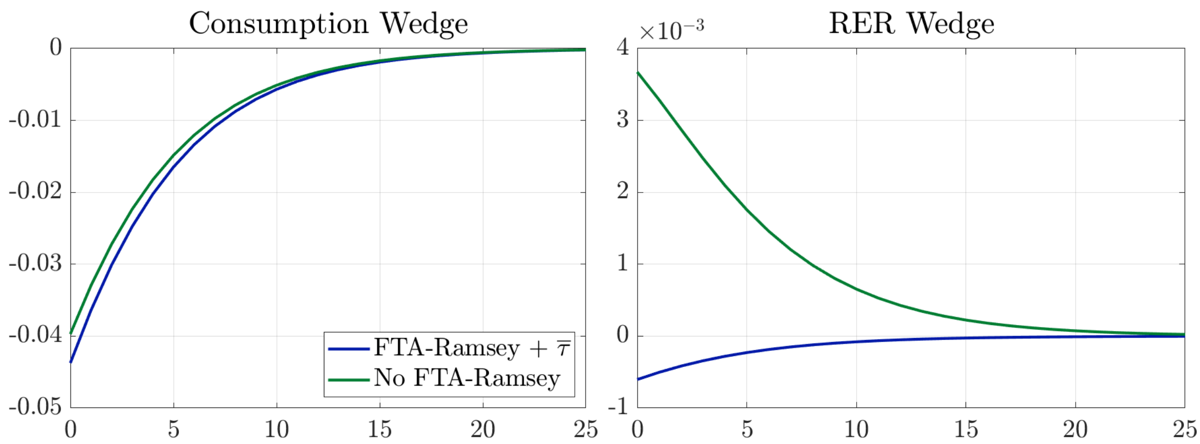
A comparison of Figures 3 and 5 clarifies the role of inter- and intra-temporal incentives in driving the interaction between trade and financial policy, as well as the relative size of capital flow taxes with and without free trade. In scenario 1, the alignment of incentives results in reinforcing consumption and RER wedges and, in turn, larger capital inflow taxes without free tree. In this sense, capital flow taxes and tariffs are complements. In contrast, in scenario 2, or more generally when inter- and intra-temporal incentives are misaligned, high import tariffs in early periods can appreciate the real exchange rate, disincentivising consumption and capital inflows in the near term, without the need for additional capital flows. In this case, the resulting

Figure 4: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Experiment 2



Notes: Time profile for macroeconomic outcomes in Experiment 2, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

Figure 5: Decomposition of Optimal Capital Flow Taxes for Scenario 2



Notes: Time profile for Home capital flow tax components in Scenario 2, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

real exchange rate moves without free trade partly substitute for capital inflow taxes.

4.6 Generality of Results

So far, we have made a number of simplifications which allow us to characterise the optimal policy sharply, but abstract from many potentially important features of standard models used in open-economy macroeconomics. Nevertheless, many of our results carry over to more general settings, which we discuss here.

Production Economies. Our baseline model is set up as an endowment economy to abstract from the complexities of price-setting and labour supply, a key focus of open-economy macroeconomics in past decades (e.g. Devereux and Engel, 2003; Benigno and Benigno, 2003; Corsetti, Dedola, and Leduc, 2010). Nevertheless, with full specialisation assumed ($y_1 = 1$, $y_2 = 0$, $y_1^* = 0$ and $y_2^* = 1$), our endowment model is isomorphic to one with production subject to technology $y_1 = f(A, L)$ and flexible prices, where each country is endowed with a fixed quantity of labour $\overline{L}$. If we further assume that the function f is first-order homogeneous in A, fluctuations in y_1 and y_2^* in our endowment economy have the same macro implications as movements in Home and Foreign productivity—A and A^*, respectively.[17] However, the full specialisation case, frequently used in international macroeconomics, does preclude cases akin to scenario 2 above. Moving away from the full specialisation case, we assume labour in each country, is employed in two sectors, one which produces good 1 and another producing good

[17] Alternatively, if technology is linear and productivity is constant, y_1 and y_2^* can reflect exogenous movements in labour supply (L and L^*, respectively) such as those studied in Guerrieri, Lorenzoni, Straub, and Werning (2020), in the context of the Covid-19 lockdowns.

2. The latter can be interpreted as an 'export' sector and fluctuations in y_2^* can represent fluctuations in export-sector productivity.

Trade Disruptions and Sanctions. While we focus on fluctuations in the path for endowments, the mechanism we describe applies to a wide range of economic fluctuations. A timely application of our results is to the case of global trade disruptions and multilateral sanctions, which can be thought of as an increase in the (iceberg) cost of imports for the planning country. Unlike a tariff, the costs are not rebated to households, nor do they re-allocate across countries.[18] Through the lens of our model, faced with trade disruptions which increase the cost of imports, the optimal policy mix will require capital inflow taxes. Moreover, even if we allow for a tariff to be optimally chosen, the size of the optimal capital flow tax will rise—consistent with our key mechanism, since inter- and intra-temporal incentives for the planner are aligned in this setting.

Concretely, suppose there are temporary and multilateral sanctions in place, which will be relaxed in the near future. The Home planner will seek to tax capital inflows to delay consumption because it is relatively expensive in the near future. Absent a FTA, the planner would also want to subsidize good 2 in the near future, partly offsetting the wedge introduced in the relative demand by the sanctions. As a result, inter- and intra-temporal incentives are aligned. Consistent with our theory, the optimal capital inflow tax rises.[19]

Country Size. This paper considers a two-country model where each country is large in both goods and financial markets. As a consequence, the planner internalises the effect of domestic allocations on both goods prices and the real interest rate, motivating the use of both capital controls and tariffs. Appendix D details a small-open economy setting (i.e. with $N \to \infty$ Foreign countries). As pointed in Costinot et al. (2014), countries remain large in goods markets for their *domestic* variety, but the ability to manipulate the world interest rates (inter-temporal margin) disappears.[20]

We show that, in the knife-edge case where $\sigma = \phi = 1$ (Cole and Obstfeld, 1991) the required size of capital controls to address inter- and intra-temporal incentives is the same. Therefore, in scenario 1, as $N \to \infty$, the optimal size of capital controls in both the FTA and no-FTA case is unchanged. However, the optimal import tariff falls (but is always non-zero), since Home goods become more scarce. Moving away from this, when $\sigma > \phi$, the size of capital controls falls as N rises. In the case $\sigma < \phi$, capital controls move in opposite directions across the two regimes, illustrating that inter- and intra-incentives are misaligned.

[18]We can interpret multilateral sanctions example as a case in which the sanctions are set by an international organisation or coalition, or a third-party country.

[19]Varying σ and ϕ indicates that, under the FTA, inter- and intra-temporal incentives are opposed. This is true when we are restricted to capital flow taxes, because capital controls cannot offset the wedge induced by sanctions.

[20]However, Egorov and Mukhin (2020) show that in the presence of nominal rigidities and dollar currency pricing, i.e. when world exports are priced in dollars, US prices affect the world stochastic discount factor and the US is able to manipulate the inter-temporal terms of trade even if it is small.

5 Optimal Strategic Planning Allocation

We now consider the case in which both countries seek to maximise domestic welfare, taking into account each others' actions. We look for a Nash equilibrium, considering each government's optimisation problem and taking the other's tax sequence $\{\theta_t^{(*)}, \tau_t^{(*)}\}$ as given, where the asterisk denotes Foreign quantities. τ_t^*, specifically, denotes foreign tariffs that are levied on good 1. In the main text, we focus on the Nash equilibrium with both capital control *and* trade tariffs being set, and we present the equilibrium with free trade in Appendix B.1.

5.1 Without Free Trade

Defining the vector of Foreign goods-specific tariffs by $\boldsymbol{\tau}_t^* \equiv [(1+\tau_t^*)^{-1} \ 1]'$, the following proposition details the implementability constraint for the Home planner.

Proposition 3 (Implementability for Nash Planner without FTA) *The Home allocation forms part of an equilibrium without an FTA if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \qquad \text{(IC-Nash-nFTA)}$$

Proof: See Appendix B.3. □

With this, the Home planning problem in the strategic setting is given by:

$$\max_{\{\mathbf{c}_t\}} \sum_{t=0}^{\infty} u(C_t) \qquad \text{(P-Nash-nFTA)}$$

$$\text{s.t.} \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \qquad \text{(IC-Nash-nFTA)}$$

$$C_t \equiv g(\mathbf{c}_t) \qquad \text{(nFTA)}$$

which contrasts with the no-FTA problem when the Foreign planner is passive (P-Unil-nFTA).

Optimal Allocation. Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu \hat{\mathcal{MC}}_{1,t}^{nFTA} \qquad (13)$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu \hat{\mathcal{MC}}_{2,t}^{nFTA} \qquad (14)$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)(1+\tau_t^*)g_1^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_1^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}}\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ \hat{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_2^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\end{aligned}$$

$$+ u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

The Foreign planner undertakes an analogous maximisation. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, which is summarised in the following proposition.

Proposition 4 (Capital Control and Tariff Wars) *In a Nash equilibrium where each country chooses optimal capital controls* $\{\theta_t, \theta_t^*\}_{t\geq 0}$ *and tariffs* $\{\tau_t, \tau_t^*\}_{t\geq 0}$*, the allocations* $\{\mathbf{c}_t, \mathbf{c}_t^*\}_{t\geq 0}$ *must satisfy*

$$\frac{\hat{\mathcal{MC}}_{1,t}^{nFTA}}{\hat{\mathcal{MC}}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{MC}}_{2,t}^{nFTA}}{\hat{\mathcal{MC}}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA} \tag{15}$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad \textit{for } i = 1, 2$$

Proof: See Appendix B.3. □

The equilibrium conditions above reflect the ratio of the marginal cost of a unit of consumption for the planner across the Home and Foreign country, for each good variety. Their interpretation is consistent with that in Section 3. The coefficients $\{\alpha_{i,0}^{nFTA}\}$ can be interpreted as the bargaining power of the Foreign country relative to the Home with respect to each good, and they depend on initial conditions.

5.2 Numerical Exercises

To analyse the Nash equilibria, it is first useful to define two quantities to capture the difference in the cost of borrowing in the Home *vis-à-vis* the Foreign country, and the relative ratio of tariffs at Home *vis-à-vis* Foreign:

$$\Delta^R = \frac{1-\theta_t}{1-\theta_t^*} \qquad \Delta^\tau = \frac{1+\tau_t}{1+\tau_t^*}$$

If $\Delta^R > 1$, then the cost of borrowing in the Home country is higher vis-à-vis the Foreign country, while $\Delta^\tau > 1$ reflects a higher *import* tariff at Home *vis-à-vis* the Foreign country.[21] The distance of these quantities from unity captures the total distortion to the inter- and intra-temporal margins, respectively.

With these definitions, we revisit the scenarios in Section 4 to assess the impact of strategic interactions on the macroeconomic allocations and policy outcomes. Figure 10 presents the

[21] $\Delta^R > 1$ can be achieved either by $\theta_t < 0$ (a Home capital inflow tax) keeping $theta_t^* = 0$, or $\theta_t^* > 0$ (a Foreign capital inflow subsidy) keeping $\theta_t = 0$, or any combination where $\theta_t < \theta_t^*$, regardless of their sign. Δ^τ is defined such that if both countries levy an equal tariff on their *respective* imports, $\Delta^\tau = 1$.

results for scenario 1, where the Home endowment of good 1 is temporarily low relative to its long-run value. The left-hand panel indicates that the Home and Foreign planners' incentives for consumption align in this setting: consumption is be delayed in the Home country, through capital inflow taxes, and brought forward in the Foreign, through capital inflow subsidies. The right-hand panel shows that there is also a similar alignment of incentives for good-1 consumption. As a result, the Home capital inflow tax tends to be smaller without free trade, since the Foreign country chooses to subsidise its own inflows by less.

Figure 7 plots the corresponding distortions from financial and trade policy. The left-hand plot indicates that the initial financial wedge, which weighs on global welfare, is higher, with $\Delta^R > 1$ on impact but approaching 1 over time as $y_{1,t}$ approaches $\overline{y}_1$. Moreover, because Scenario 1 involves a fall in the variety of Home's preferred good, the home tariff dominates and $\Delta^\tau < 1$ on impact, rising thereafter.[22]

Next, we look at capital control and tariff interactions.[23] In response to a good-1 downturn at Home, capital inflow taxes are larger absent a FTA in the strategic setting, as the left-hand panel of Figure 10 shows. This is consistent with the policy interactions discussed in the unilateral setting.

However, unlike in the unilateral case, the same is true for Scenario 2. Because the Foreign country tariff is very large— taxing c_1^* heavily in the early periods—there is increased pressure on the exchange rate to depreciate. As a result, the Home planner must levy a larger inflow tax to implement any given path for aggregate consumption.[24] The same results hold for the Foreign country.

Comparative Statics. At the benchmark calibration ($\sigma = 2, \phi = 1.5$), countries engage in competition over both capital controls and trade tariffs leading to $\Delta^R, \Delta^\tau \neq 1$. As the elasticity of inter-temporal substitution $\frac{1}{\sigma}$ falls though—i.e. σ rises—countries levy larger capital controls in an attempt to reallocate consumption inter-temporally. This results in a higher $|\Delta^R|$. When σ is high, a representative household is more insensitive to change in the interest rate when choosing to allocate consumption across periods. In contrast, as $\frac{1}{\sigma}$ rises, households are more sensitive to changes in the interest rate and smaller capital controls are levied.

Conversely, when the trade elasticity ϕ is low, countries engage more in a tariff war leading to a higher $|\Delta^\tau|$. This reflects the well-understood result in public finance that a planner optimally chooses to tax commodities for which demand is price-inelastic. This *inverse-elasticity* result extends to a 'policy war' style setting, involving competition in capital flow taxes and import tariffs (see, for example, Chari and Kehoe, 1999).[25]

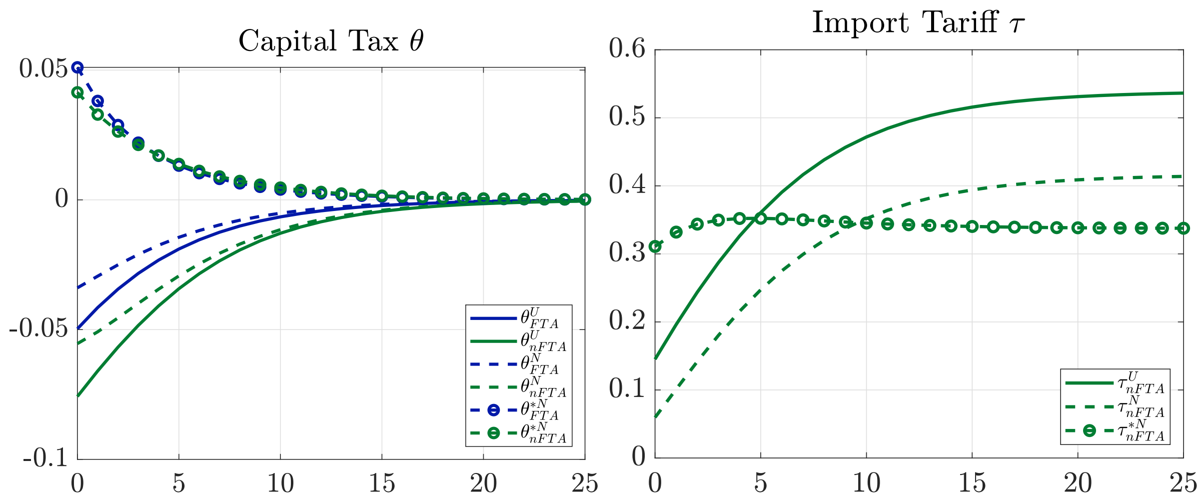
[22]In Scenario 2 the result is analogous. The aggregate distortion is larger, even though individual countries' capital flows are smaller.

[23]It is worth noting that in Scenario 1, while the Home country's inter- and intra-temporal incentives are aligned, they are opposed for the Foreign, and vice versa for Scenario 2.

[24]These results generalise to the case where aggregate endowments are allowed to vary.

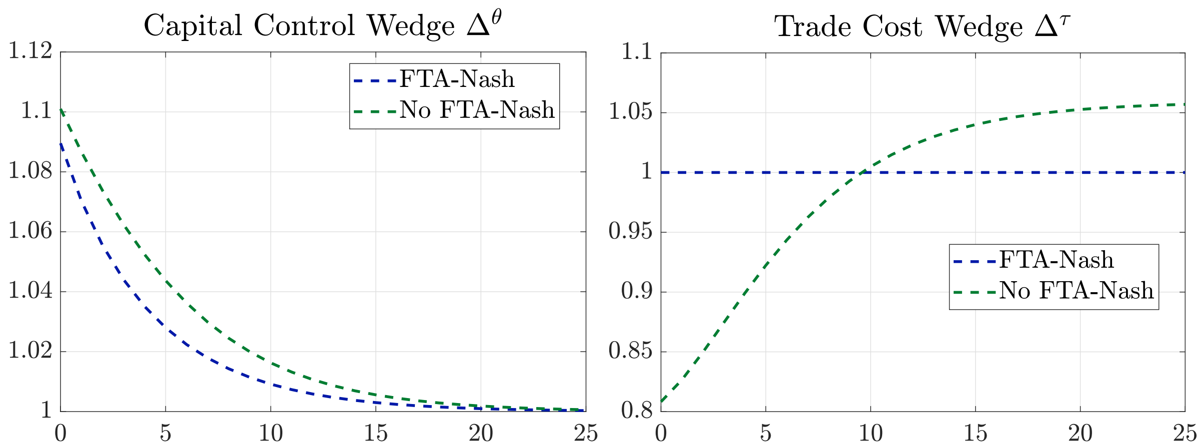
[25]These findings are consistent with the Arrow-Debreu approach of relabelling the future delivery of commodities as a separate good.

Figure 6: Optimal Capital Inflow Taxes and Import Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 1



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home). 'N' denotes Nash outcome.

Figure 7: Capital Control and Tariff Weds in the Nash Equilibrium in scenario 1



Notes: Difference in cost of borrowing and tariffs across countries.

6 Welfare and International Spillovers

Finally, we assess the consequences and spillovers of policy in terms of welfare. Does the optimal policy simply reallocate from the Foreign country to the Home, or does it contribute to increase Home welfare at a disproportional cost to Foreign welfare, and therefore world welfare? What are the costs of capital control wars, and are policy wars costlier when a FTA is not in place?

First, we consider the cooperative problem where consumption allocations are chosen to maximise joint (world) welfare. The cooperative planning problem is given by:

$$\max_{\{\mathbf{c}_t\}} \sum_{t=0}^{\infty} \beta^t \Big[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}_t^*)) \Big], \tag{P-Coop}$$

$$\text{s.t} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \tag{RC}$$

$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C) \tag{FTA}$$

where κ is the weight attributed to Foreign welfare.

The following Proposition summarises the key property of the global cooperative problem.

Proposition 5 (Globally Cooperative Allocation) *In the cooperative allocation, no intervention is optimal such that, if $\kappa = 1$, $\theta_t = \tau_t = 0$.*

Proof: See Appendix C.1. □

Moreover, this leads to the following Corollary:

Corollary (Negative Spillovers) *Any policy intervention which improves Home welfare necessarily reduces global welfare.*

Proof: Follows directly by combining Propositions 2 and 5. □

Table 2 reports the difference in present discounted welfare in scenarios 1 and 2 under the optimal policy, relative to the decentralised allocation, for the Home and Foreign representative agents respectively. Our results confirm that capital and goods taxes are distortionary and do not simply reallocate consumption across borders. In the case where the Foreign country is passive, the costs to the Foreign country outweigh the gains in the Home country, resulting in a loss in global welfare. In the presence of a FTA, capital controls change in the path of consumption over time, in a manner that is inefficient for the Foreign country. In a Nash equilibrium with free trade, the Foreign country benefits relative to the unilateral case by levying taxes itself, but global welfare ultimately falls further.

In the absence a FTA, countries levy taxes not only to change the path of consumption over time, but also its composition across goods varieties. So the the welfare costs from policy wars are higher when countries engage in both capital controls and tariff wars. As the elasticity of inter-temporal substitution rises, welfare costs from capital control wars under FTA become very small but are almost unchanged absent an FTA. In contrast, costs sharply fall as ϕ rises

Table 2: Welfare and Spillovers. Welfare expressed in terms of % consumption equivalent variation ($-ve$ implies welfare gain).

	H	F	Global $\sum_{H,F}$
scenario 1			
FTA (Unilateral)	-0.13	0.23	0.050
without FTA (Unilateral)	-0.22	0.27	0.025
with FTA (Nash)	0.068	0.067	0.068
without FTA (Nash)	1.71	1.58	1.65
scenario 2			
with FTA (Unilateral)	-0.061	0.011	0.0027
without FTA (Unilateral)	-0.082	0.39	0.15
with FTA (Nash)	0.16	-0.0007	0.080
without FTA (Nash)	5.2	0.93	3.1

both with and without an FTA in place. Therefore, the costs to *both* capital control and tariff wars predominantly arise from intra-temporal choice distortions.

7 Conclusion

In this paper, we provide a unified framework for the analysis of capital-flow taxation and trade tariffs. We emphasise that financial and trade policy are tightly interlinked and we show that introducing tariffs without adjusting capital inflow taxation can come at significant costs to efficiency. Specifically, time-variation in tariffs can lead to over- or under-borrowing by households due to their effects on the path for real exchange rates. We show that whether capital controls are larger or smaller in the absence of a FTA depends on the state of the economy and specifically, on whether the inter- and intra-temporal incentive to manipulate the terms of trade are aligned.

In a Nash equilibrium, where there is retaliation across countries, capital controls tend to be larger absent a FTA in all states of the economy because of the effect of tariff wars on the real exchange rate. Moreover, global welfare falls significantly more when tariffs wars exist in conjunction with capital control wars. Turning to welfare, while employing tariffs in addition to capital controls can improve welfare domestically, this comes at a disproportionate cost to foreign welfare

We emphasise two directions for research in future work, relating to the role of incomplete markets and balance-sheet effects. First, while in complete markets there is no scope for policy to improve the cooperative allocation absent additional frictions, this is not true with incomplete markets. Second, the interaction between capital flow taxes and tariffs will depend on the currency denomination of debt. In a nominal model, foreign currency debt alters the inter- and intra-temporal incentives facing the planner, for a given path of endowments, due to the incentive the inflate away debt obligations.

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Appendix

A Derivations and Proofs

A.1 Pareto Frontier

This sub-section provides derivations for the Pareto frontier, which is defined in Section 2. The Pareto frontier summarises combinations of consumption allocations $\{c_{1,t}, c_{2,t}\}$ which are Pareto efficient, given a level of aggregate consumption C_t.

The Home representative household chooses their consumption by minimising expenditure, for a given level of aggregate consumption $\overline{C}$:

$$\min_{c_{1,t}, c_{2,t}} \quad p_{1,t}c_{1,t} + p_{2,t}c_{2,t} \quad \text{s.t.} \quad \overline{C} = g(\mathbf{c}_t)$$

The first-order conditions for this problem yield the Home relative demand equation:

$$\frac{g_{1,t}}{g_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c_{2,t}}{c_{1,t}}\right)^{\frac{1}{\phi}} \tag{16}$$

where $p_{1,t}/p_{2,t} \equiv 1/TOT_t$ and TOT_t refers to the terms of trade.

To derive the Pareto frontier, note that the analogous Foreign relative demand curve is:

$$\frac{g^*_{1,t}}{g^*_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c^*_{2,t}}{c^*_{1,t}}\right)^{\frac{1}{\phi}} \tag{17}$$

Equating relative prices across countries, equations (16) and (17) yield:

$$\frac{c^*_{2,t}}{c^*_{1,t}} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{c_{2,t}}{c_{1,t}} \tag{18}$$

This expression for optimal relative consumption must be consistent with goods market clearing ($Y_{i,t} = c_{i,t} + c^*_{i,t}$ for $i = 1, 2$). Combining (18) with goods market clearing, we attain the following expressions for consumption:

$$c_{1,t} = \frac{bc_{2,t}Y_{1,t}}{Y_{2,t} - (1-b)c_{2,t}} \tag{19}$$

$$c_{2,t} = \frac{c_{1,t}Y_{2,t}}{bY_{1,t} + (1-b)c_{1,t}} \tag{20}$$

where $b \equiv \left(\frac{\alpha}{1-\alpha}\right)^2$.

Solving for $\mathrm{d}c_i(C)/\mathrm{d}C$ Rearranging the Armington aggregator, we can show that:

$$c_{1,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}}{\alpha^{\frac{1}{\phi}}} \right]^{\frac{\phi}{\phi-1}} \tag{21}$$

$$c_{2,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}}}{(1-\alpha)^{\frac{1}{\phi}}} \right]^{\frac{\phi}{\phi-1}} \tag{22}$$

Equating equations (20) with (22) yields:

$$\left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} (bY_{1,t} + (1-b)c_{1,t}(C_t)) = c_{1,t}(C_t) Y_{2,t} (1-\alpha)^{\frac{1}{\phi-1}}$$

Totally differentiating this expression and rearranging yields:

$$\frac{\mathrm{d}c_{1,t}(C_t)}{\mathrm{d}C_t} = \frac{C_t^{-\frac{1}{\phi}} (1-\alpha)^{-\frac{1}{\phi}} c_{2,t}^{\frac{1}{\phi}} (bY_{1,t} + (1-b)c_{1,t}(C_t)}{Y_{2,t} - c_{2,t}(C_t)(1-b) + \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{-\frac{1}{\phi}} (1-\alpha)^{-\frac{1}{\phi}} c_{2,t}^{\frac{1}{\phi}} (bY_{1,t} + (1-b)c_{1,t}C_t)}$$

The expression for $\mathrm{d}c_{2,t}(C_t)/\mathrm{d}C_t$ can be derived analogously.

A.2 Foreign Household Optimisation

This sub-section details the representative Foreign consumer's optimisation problem, which acts as a constraint for the unilateral Home Ramsey planner in Section **??**.

Foreign households maximise their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices $\mathbf{p}_t$:

$$\max_{\{\mathbf{c}_t\}} \quad U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t))$$
$$\text{s.t.} \quad \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0$$

The first-order conditions for this problem are given by (3) and (4) in Section **??**, where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint.

A.3 Proof to Proposition 2

First, note that any outcome achievable in (P-Unil-FTA) is achievable in (P-Unil-nFTA). Part (i) follows immediately since (P-Unil-nFTA) is a relaxed version of (P-Unil-FTA) therefore the planner achieves weakly better outcomes when the FTA is relaxed. However, we analyse this further. Equations (5), (6), and (7) satisfy the following total derivative rule:

$$\frac{d\mathcal{L}}{dC} = \frac{\partial\mathcal{L}}{\partial c_1}c_1'(C) + \frac{\partial\mathcal{L}}{\partial c_2}c_2'(C)$$

The solution to (P-Unil-FTA) (when an FTA is in force) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C), c_2'(C)$ are positive and increasing functions in Appendix A.1, generally $sign(\frac{d\mathcal{L}}{dc_1}) = -sign(\frac{d\mathcal{L}}{dc_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained optimal allocation.

In contrast, the solution to (P-Unil-nFTA) given by (6) and (7) implies $\frac{d\mathcal{L}}{dc_1} = \frac{d\mathcal{L}}{dc_2} = 0$ which necessarily implies aggregate consumption is (unconstrained) optimal as well. Formally, denote,

$$\overline{C} = \{C : \max \quad \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto Frontier}\}, \tag{23}$$

where $\overline{C}$ is a scalar because $\mathcal{L}$ is strictly concave in the region of interest. Then note that $\frac{d\mathcal{L}}{dc_1}_{|c_H(\overline{C}),c_2(\overline{C})}, \frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, e.g. $\frac{d\mathcal{L}}{dc_1}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred.

Furthermore, (ii) follows since it must be then that $c_1'(C)$, $c_2'(C)$ implied by (6) and (7) violate Lemma 1 (ii) if $\frac{d\mathcal{L}}{dc_1}_{|c_1(\overline{C}),c_2(\overline{C})}, \frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. Conversely, if $\frac{d\mathcal{L}}{dC} = 0 \implies \frac{\partial\mathcal{L}}{\partial c_1} = 0, \frac{\partial\mathcal{L}}{\partial c_2} = 0$ if $c_1'(C), c_2'(C)$ are not binding, i.e. the constraints are identical to the correspondence implied by (6) and (7).

(iii) The allocations coincide when there is no trade in goods in equilibrium as the households' choice is optimal for the planner. □

A.4 Derivatives of the Consumption Aggregator

In this sub-section, we define the derivatives of the Armington (1969) aggregator which defines aggregate consumption in our computational scenarios. We present the expressions for the representative Home consumer only, but they are analogous for the representative Foreign consumer.

The first derivatives of the Home aggregator are given by:

$$g_1(\mathbf{c}_t) \equiv \frac{\partial g(\mathbf{c}_t)}{\partial c_{1,t}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

$$g_2(\mathbf{c}_t) = \frac{\partial g(\mathbf{c}_t)}{\partial c_{2,t}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

The second derivatives are:

$$g_{11}(\mathbf{c}_t) = -\frac{1}{\phi}\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}$$

$$
\begin{aligned}
&+ \frac{1}{\phi}\alpha^{\frac{2}{\phi}} c_{1,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{12}(\mathbf{c}_t) =& \frac{1}{\phi}\alpha^{\frac{1}{\phi}}(1-\alpha)^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{21}(\mathbf{c}_t) =& g_{12}(\mathbf{c}_t) \\
g_{22}(\mathbf{c}_t) =& -\frac{1}{\phi}(1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}(1-\alpha)^{\frac{2}{\phi}} c_{2,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}}
\end{aligned}
$$

A.5 Comparative Statics

Within the model, two parameters are particularly important for governing the size of the planner's intra- and inter-temporal incentives to manipulate the terms of trade: the intra-temporal elasticity of substitution between goods ϕ (i.e. the trade elasticity) and the coefficient of relative risk aversion σ (i.e. the inverse inter-temporal elasticity of substitution). In doing so, these parameters influence the size of both the optimal capital inflow taxes and optimal import tariffs. They do so in a manner that is inversely related to the elasticity: the lower the elasticity, the higher the taxes, and vice versa.

Figure 8 demonstrates this for the inter-temporal trade elasticity in the content of scenario 1—although the 'inverse elasticity rule' holds in both scenarios. As the right-hand figure shows, optimal import tariffs are both larger and vary more over time when the trade elasticity is lower. These intra-temporal incentives interact with the optimal capital flow taxes too, which are higher for lower trade elasticities, regardless of the prevailing trade agreement.

Similarly, Figure 9 shows that optimal capital flow taxes are larger when the inter-temporal elasticity of substitution is lower (i.e. higher coefficient of relative risk aversion σ). In turn, variation in import tariffs is larger when σ is high.

B Nash Allocation

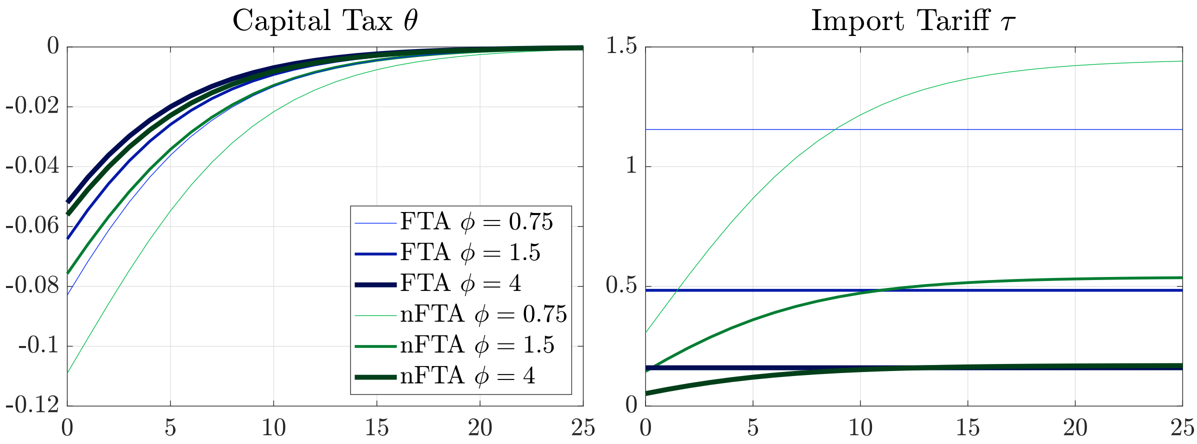
B.1 With Free Trade

Focusing on the Home planning problem, we can characterise the optimal allocation with a FTA in place, taking the sequence of Foreign capital flow taxes $\{\theta_t^*\}$ as given. Faced with these taxes, the Foreign Euler equations, for $i = 1, 2$ can be written:

$$
u^{*\prime}(C_t^*) g_i^*(\mathbf{c}_t^*) = \beta(1-\theta_t^*)(1+r_{i,t}) u^{*\prime}(C_{t+1}^*) g_i^*(\mathbf{c}_t^*) \tag{24}
$$

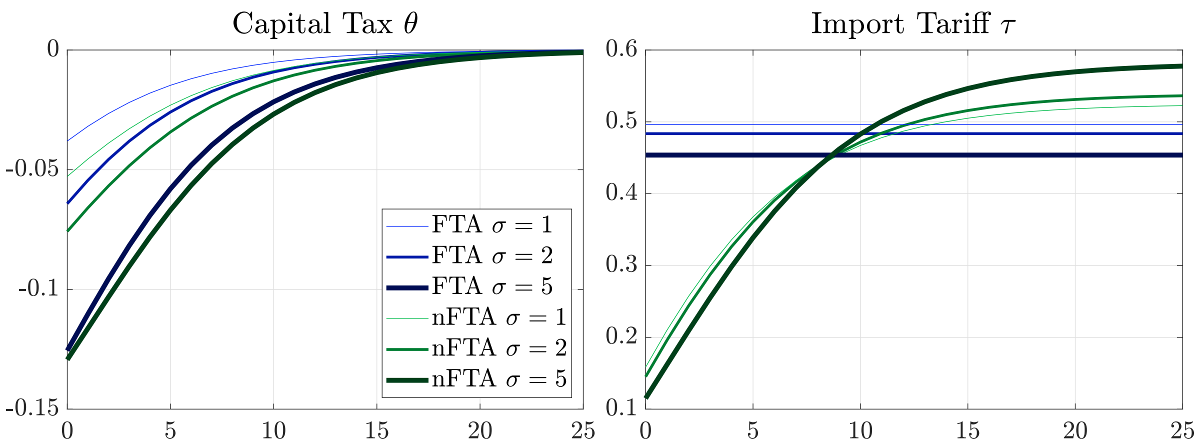
These Foreign optimality conditions, the Home inter-temporal budget constraint and the market clearing conditions yield an implementability condition for the Home planner, which is described

Figure 8: Comparative Statics of Optimal Capital Flow Taxes and Tariffs with Respect to the Intra-temporal Trade Elasticity ϕ in scenario 1



Notes: Time profile for Home capital flow tax and import tariff in scenario 1, simulated for 50 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

Figure 9: Comparative Statics of Optimal Capital Flow Taxes and Tariffs with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in scenario 1



Notes: Time profile for Home capital flow tax and import tariff in scenario 1, simulated for 50 periods, with three different values of the coefficient of relative risk aversion σ (i.e. inverse inter-temporal elasticity of substitution. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

in the following proposition.

Proposition 3A (Implementability for Nash Planner with FTA) *Since* $1 + r_{i,t} \equiv p_{i,t}/p_{i,t+1}$*, when the Foreign country seeks to set* $\{\mathbf{c}_t^*\}$ *in order maximise domestic welfare, then the Home allocation* $\{\mathbf{c}_t\}$ *forms part of an equilibrium if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-FTA}$$

The Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \tag{IC-Nash-FTA}$$

$$\mathbf{c}_t=\mathbf{c}(C_t), \quad \mathbf{c}_t^*=\mathbf{c}^*(C_t) \tag{FTA}$$

which is comparable to the unilateral problem (P-Unil-FTA), albeit with an additional term in the implementability constraint reflecting the Foreign capital flow tax θ_t^*.

Optimal Allocation. Problem (P-Nash-FTA) yields the optimality condition:

$$u'(C_t)=\mu\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\hat{\mathcal{MC}}_t^{FTA} \tag{25}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{\mathcal{MC}}_t^{FTA} \equiv\, & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot\mathbf{c}'(C_t)+u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t)\nabla g^*(\mathbf{c}^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ & +u^{*\prime}(C_t^*)\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial C_t}\cdot[\mathbf{c}_t-\mathbf{y}_t]\end{aligned}$$

Taking the ratio of t and $t+1$ optimality conditions further implies that:

$$\frac{u'(C_t)}{u'(C_{t+1})}=\frac{1}{1-\theta_t^*}\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_{t+1}^{FTA}} \tag{26}$$

Combining equation (26) with the Foreign Euler equations (24) and the analogous Home Euler equations, yields an expression for $1-\theta_t$. The planning problem of the Foreign government is symmetric, so an analogous expression for $1-\theta_t^*$ can be derived. After some simplification, the combination of these expressions yields a mutual best response function, given by:

$$\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_t^{*FTA}} = \alpha_0^{FTA} \tag{27}$$

where

$$\alpha_0^{FTA} \equiv \frac{\hat{\mathcal{MC}}_0^{FTA}}{\hat{\mathcal{MC}}_0^{*FTA}}$$

This is the strategic counterpart of equation (5) in Section ??. In the Nash bargaining setup, α_0^{FTA} can be interpreted as the bargaining power of the Foreign country relative to the Home.

B.2 Derivation of Strategic Planning Allocation

Consider the problem faced by the Foreign planner,

$$\max_{\{\mathbf{c}_t^*\}} \sum_{t=0}^{\infty} \beta^t \ u\left(g(\mathbf{c}_t^*)\right) \tag{$P1^{*\ Nash}$}$$

$$s.t \quad \sum_{t=0}^{\infty} \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \beta^t u'(g(\mathbf{c}_t))\boldsymbol{\tau}_t^{-1} \boldsymbol{\nabla} g(\mathbf{c}_t) \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0 \tag{$IC^{*\ Nash}$}$$

where,

$$\boldsymbol{\tau}_t = \begin{bmatrix} 1 & 0 \\ 0 & (1-\tau_t) \end{bmatrix} \tag{28}$$

The first order conditions for the Foreign country with respect to $c_{H,t}^*$ and $c_{F,t}^*$ are given by,

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu\left[\Pi_{s=0}^{t-1}(1-\theta_s)\right]\left\{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t}\left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array}\right] - C_t^{-\sigma}\left[\begin{array}{c} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array}\right]\right\}, \tag{29}$$

$\implies$

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu\ \hat{MC}_{1,t}^*$$

and,

$$C_t^{*\ -\sigma} g_{2,t}^* = \mu\left[\Pi_{s=0}^{t-1}(1-\theta_s)\right]\left\{C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t}\left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array}\right] - C_t^{-\sigma}\left[\begin{array}{c} g_{12,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{22,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array}\right]\right\} \tag{30}$$

$\implies$

$$C_t^{-\sigma} g_{2,t}^* = \mu\ \hat{MC}_{2,t}^*$$

B.3 Proof to Proposition 4

Dividing (13) by its $t+1$ analogue yields,

$$\frac{C_t^{-\sigma} g_{1,t}}{C_{t+1}^{-\sigma} g_{1,t+1}} = \frac{1}{1-\theta_t^*} \frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}} \tag{31}$$

Evaluating the Foreign analogue for $i = 1$, i.e. (29), and using it to substitute out $\frac{1}{1-\theta_t^*}$ above, and using the analogous Home euler to substitute in $1-\theta_t$ yields the expression for the optimal tax on capital flows levied by the Home country:

$$1-\theta_t = \frac{1+\sigma C_t^{*-1}\begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - \frac{1}{g_{1,t}^*}\begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix}}{1+\sigma C_{t+1}^{*-1}\begin{bmatrix} g_{1,t+1}^*(c_{1,t+1}-y_{1,t+1})+ \\ g_{2,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1}-y_{2,t+1}) \end{bmatrix} - \frac{1}{g_{1,t+1}^*}\begin{bmatrix} g_{11,t+1}^*(c_{1,t+1}-y_{1,t+1})+ \\ g_{21,t+1}^*(1-\tau_{t+1}^*)^{-1}(c_{2,t+1}-y_{2,t+1}) \end{bmatrix}} \tag{32}$$

Dividing (29) by its $t+1$ analogue yields,

$$\frac{C_t^{*\ -\sigma} g_{1,t}^*}{C_{t+1}^{*\ -\sigma} g_{1,t+1}^*} = \frac{1}{1-\theta_t} \frac{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}{C_{t+1}^{-\sigma} g_{1,t+1} + \sigma C_{t+1}^{-\sigma-1} g_{1,t+1}\begin{bmatrix} g_{1,t+1}(c_{1,t+1}^*-y_{1,t+1}^*)+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^*-y_{2,t+1}^*) \end{bmatrix} - C_{t+1}^{-\sigma}\begin{bmatrix} g_{11,t+1}(c_{1,t+1}^*-y_{1,t+1}^*)+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^*-y_{2,t+1}^*) \end{bmatrix}}$$
$$= \frac{1}{1-\theta_t} \frac{\hat{MC}_{1,t}^*}{\hat{MC}_{1,t+1}^*} \tag{33}$$

and following the analogous steps as for (31) yields the expression for the optimal tax on capital flows levied by the Foreign country:

$$1-\theta_t^* = \frac{1+\sigma C_t^{-1}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - \\ \dfrac{1}{g_{1,t}}\begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}{1+\sigma C_{t+1}^{-1}\begin{bmatrix} g_{1,t+1}(c_{1,t+1}^*-y_{1,t+1}^*)+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^*-y_{2,t+1}^*) \end{bmatrix} - \\ \dfrac{1}{g_{1,t+1}}\begin{bmatrix} g_{11,t+1}(c_{1,t+1}^*-y_{1,t+1}^*)+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c_{2,t+1}^*-y_{2,t+1}^*) \end{bmatrix}} \tag{34}$$

To reach the conditions characterizing allocations in a Nash equilibrium, combine (31) and (34) yields,

$$\frac{C_t^{*-\sigma}g_{1,t}^* + \sigma C_t^{*-\sigma-1}g_{1,t}^*\begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - \\ C_t^{*-\sigma}\begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{-\sigma}g_{1,t} + \sigma C_t^{-\sigma-1}g_{1,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - \\ C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}} = \alpha_{1,0},$$

Similarly, combining (33) and (32) yields,

$$\frac{C_t^{*-\sigma}g_{2,t}^*(1-\tau_t^*)^{-1} + \sigma C_t^{*-\sigma-1}g_{2,t}^*\begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix} - \\ C_t^{*-\sigma}\begin{bmatrix} g_{21,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*(1-\tau_t^*)^{-1}(c_{2,t}-y_{2,t}) \end{bmatrix}(1-\tau_t)^{-1}}{C_t^{-\sigma}g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1}g_{2,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - \\ C_t^{-\sigma}\begin{bmatrix} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t}(1-\tau_t)(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}(1-\tau_t^*)^{-1}} \frac{1-\tau_t^*}{1-\tau_t} = \alpha_{2,0},$$

The constant $\alpha_{1,0}$ is given by,

$$\alpha_{1,0} = \frac{\begin{array}{c} C_0^{*-\sigma} g_{1,0}^* + \sigma C_0^{*-\sigma-1} g_{1,0}^* \begin{bmatrix} g_{1,0}^*(c_{1,0} - y_{1,0}) + \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{bmatrix} - \\ C_0^{*-\sigma} \begin{bmatrix} g_{11,0}^*(c_{1,0} - y_{1,0}) + \\ g_{21,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{bmatrix} \end{array}}{\begin{array}{c} C_0^{-\sigma} g_{1,0} + \sigma C_0^{-\sigma-1} g_{1,0} \begin{bmatrix} g_{1,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^* - y_{2,0}^*) \end{bmatrix} - \\ C_0^{-\sigma} \begin{bmatrix} g_{11,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{21,0}(1-\tau_0)^{-1}(c_{2,0}^* - y_{2,0}^*) \end{bmatrix} \end{array}}$$

and $\alpha_{2,0}$ is given by,

$$\alpha_{2,0} = \frac{1-\tau_0^*}{1-\tau_0} \frac{\begin{array}{c} C_0^{*-\sigma} g_{2,0}^*(1-\tau_0^*)^{-1} + \sigma C_0^{*-\sigma-1} g_{2,0}^* \begin{bmatrix} g_{1,0}^*(c_{1,0} - y_{1,0}) + \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{bmatrix} - \\ C_0^{*-\sigma} \begin{bmatrix} g_{12,0}^*(c_{1,0} - y_{1,0}) + \\ g_{22,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{bmatrix} (1-\tau_0)^{-1} \end{array}}{\begin{array}{c} C_0^{-\sigma} g_{2,0}(1-\tau_t)^{-1} + \sigma C_0^{-\sigma-1} g_{2,0} \begin{bmatrix} g_{1,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^* - y_{2,0}^*) \end{bmatrix} - \\ C_0^{-\sigma} \begin{bmatrix} g_{12,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{22,0}(1-\tau_0)(c_{2,0}^* - y_{2,0}^*) \end{bmatrix} (1-\tau_0^*)^{-1} \end{array}}$$

Finally, substituting out τ_t and τ_t^* yields,

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t} - y_{1,t}) + \\ g_{1,t}^* S_t(c_{2,t} - y_{2,t}) \end{bmatrix} - \\ C_t^{*-\sigma} \begin{bmatrix} g_{11,t}^*(c_{1,t} - y_{1,t}) + \\ g_{21,t}^* \dfrac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t} - y_{2,t}) \end{bmatrix} \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{1,t} S_t(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - \\ C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t} \dfrac{g_{1,t}}{g_{2,t}} S_t(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \end{array}} = \alpha_{1,0},$$

and,

$$\frac{C_t^{*-\sigma} g_{2,t}^* + \sigma C_t^{*-\sigma-1} g_{2,t}^* \begin{bmatrix} g_1^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*\frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t})\end{bmatrix}}{C_t^{-\sigma} g_{2,t} + \sigma C_t^{-\sigma-1} g_{1,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*)\end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{12,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{22,t}\frac{g_{1,t}}{g_{2,t}}S_t(c^*_{2,t}-y^*_{2,t})\end{bmatrix}} \frac{g_{1,t}}{g_{2,t}}\frac{g_{2,t}^*}{g_{1,t}^*} = \alpha_{2,0},$$

which complete the proof. □

To derive the optimal tariffs, divide the Foreign by the Home optimality condition for good 1 and use the Euler to substitute in the Home optimal tariff on the LHS. Use the foreign Euler to substitute out the Foreign optimal tariff:

$$1-\tau_t = \frac{1}{S_t}\frac{C_t^{*-\sigma} g_{1,t}^* S_t + \sigma C_t^{*-\sigma-1} g_{2,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*\frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t})\end{bmatrix}}{C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^*\frac{g_{1,t}^*}{g_{2,t}^*} S_t(c_{2,t}-y_{2,t})\end{bmatrix}}$$

and,

$$1-\tau_t^* = \frac{1}{S_t}\frac{C_t^{-\sigma} g_{1,t} S_t + \sigma C_t^{-\sigma-1} g_{2,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*)\end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{12,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{22,t}\frac{g_{1,t}}{g_{2,t}}S_t(c^*_{2,t}-y^*_{2,t})\end{bmatrix}}{C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*)\end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}\frac{g_{1,t}}{g_{2,t}}S_t(c^*_{2,t}-y^*_{2,t})\end{bmatrix}}$$

abroad.

B.4 Nash equilibrium with FTA

Consider the Nash problem when a FTA is in place for both Home and Foreign planners. If a FTA is in place, $\boldsymbol{\tau}_t, \boldsymbol{\tau}_t^* = 1$, the Home planner chooses C_t and the Foreign C_t^* and $\mathbf{c}(C_t), \mathbf{c}^*(C_t^*)$ are given by Lemma 1. Then the allocations C_t, C_t^* in a Nash equilibrium must satisfy,

$$\frac{\begin{array}{c} C_t^{*-\sigma}(g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -\sigma-1} C_t^{*\prime}(C_t) \left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,t} - y_{2,t})\right] + \\ C_t^{*-\sigma}\left[(g_{11,t}^* + g_{21,t}^*) c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + g_{22,t}^*) c_{2,t}'(C_t)(c_{2,t} - y_{2,t})\right] \end{array}}{\begin{array}{c} C_t^{-\sigma}(g_{1,t} c_{1,t}^{*\prime}(C_t) + g_{2,t} c_{2,t}^{*\prime}(C_t)) + \\ \sigma C_t^{-\sigma-1} C_t'(C_t^*) \left[g_{1,t}(c_{1,t}^* - y_{1,t}^*) + g_{2,t}(c_{2,t}^* - y_{2,t}^*)\right] + \\ C_t^{-\sigma}\left[(g_{11,t} + g_{21,t}) c_{1,t}^{*\prime}(C_t^*)(c_{1,t}^* - y_{1,t}^*) + (g_{12,t} + g_{22,t}) c_{2,t}^{*\prime}(C_t^*)(c_{2,t}^* - y_{2,t}^*)\right] \end{array}} = \alpha_0^{FTA} \tag{35}$$

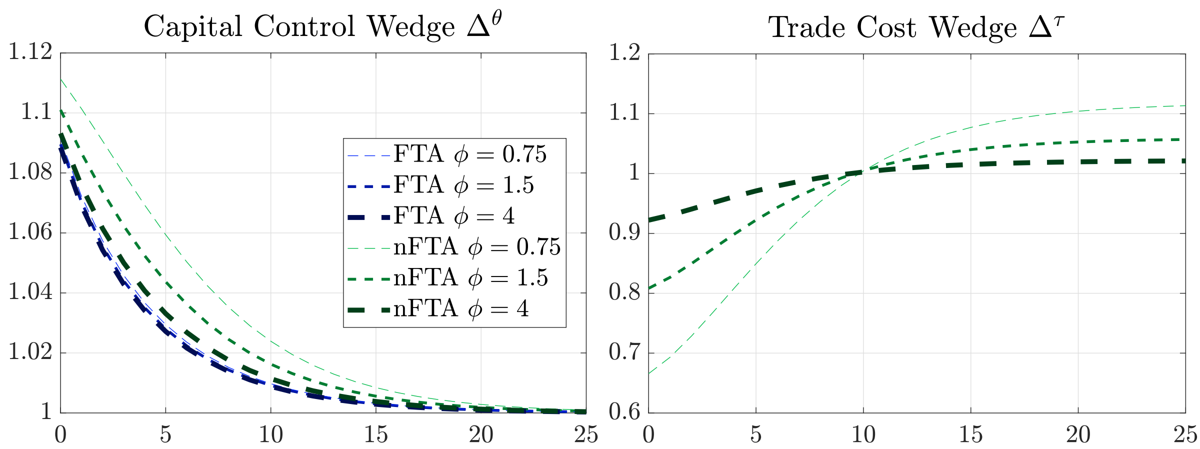
Optimal capital controls levied by the home country are given by,

$$1 - \theta_t = \frac{\begin{array}{c} (g_{1,t}^* c_{1,t}'(C_t) + g_{2,t}^* c_{2,t}'(C_t)) + \\ \sigma C_t^{*\ -1} C_t^{*\prime}(C_t) \left[g_{1,t}^*(c_{1,t} - y_{1,t}) + g_{2,t}^*(c_{2,} - y_{2,t})\right] + \\ \left[\begin{array}{c} (g_{11,t}^* + g_{21,t}^*) c_{1,t}'(C_t)(c_{1,t} - y_{1,t}) + (g_{12,t}^* + \\ g_{22,t}^*) c_{2,t}'(C_t)(c_{2,t} - y_{2,t}) \end{array}\right] \end{array}}{\begin{array}{c} (g_{1,t+1}^* c_{1,t+1}'(C_{t+1}) + g_{2,t+1}^* c_{2,t+1}'(C_{t+1})) + \\ \sigma C_{t+1}^{*\ -1} C_{t+1}^{*\prime}(C_{t+1}) \left[g_{1,t+1}^*(c_{1,t+1} - y_{1,t+1}) + g_{2,t+1}^*(c_{2,t+1} - y_{2,t+1})\right] + \\ \left[\begin{array}{c} (g_{11,t+1}^* + g_{21,t+1}^*) c_{1,t+1}'(C_{t+1})(c_{1,t+1} - y_{1,t+1}) + \\ (g_{12,t+1}^* + g_{22,t+1}^*) c_{2,t+1}'(C_{t+1})(c_{2,t+1} - y_{2,t+1}) \end{array}\right] \end{array}} \tag{36}$$

with an analogous condition for the foreign.

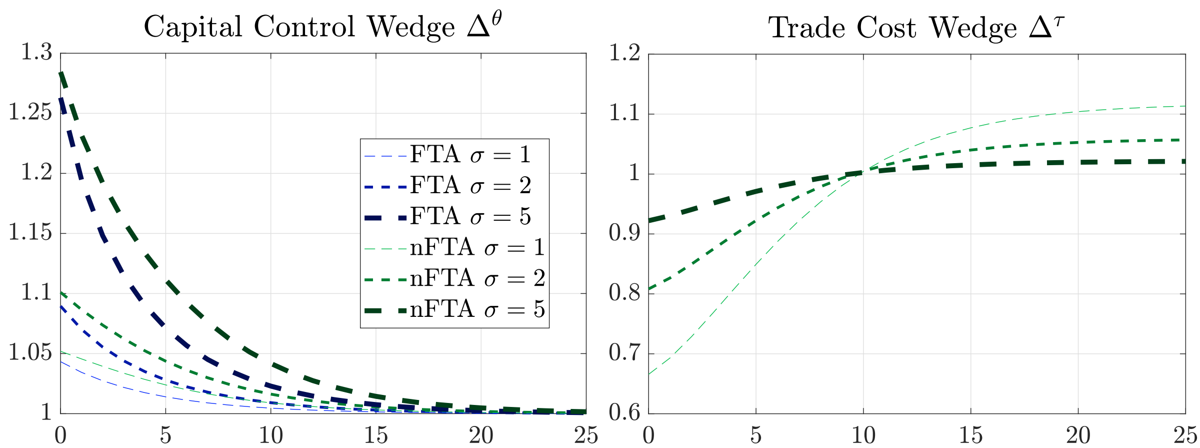
B.5 Comparative Statics: Nash

Figure 10: scenario 1: Comparative Statics with respect to ϕ



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home). 'N' denotes Nash outcome.

Figure 11: scenario 1: Comparative Statics with respect to σ



Notes: Difference in cost of borrowing and tariffs across countries.

C Cooperative Allocation

C.1 Proof to Proposition 6

When a FTA is in place, the optimal cooperative allocation satisfies,

$$u'(g(\mathbf{c}_t)) + \kappa u'(g(\mathbf{c}_t^*))\frac{dC^*}{dC} = 0 \tag{37}$$

where $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$, yielding the decentralised risk sharing condition (10) with $\kappa = \frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}_{t-1}^*))}\frac{P_{t-1}^*}{P_{t-1}}$ implying $\theta_t = 0$. Relaxing the FTA does not change the optimal allocation (since goods taxes are zero at the optimal). With FTA, first order condition follows straightforwardly by substituting $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$.

Relaxing the FTA, we get two first order conditions,

$$u'(g(\mathbf{c}_t))g_1 + \kappa u'(g(\mathbf{c}_t^*))g_1^*\frac{dc_1^*}{dc_1} = 0, \tag{38}$$

$$u'(g(\mathbf{c}_t))g_2 + \kappa u'(g(\mathbf{c}_t^*))g_2^*\frac{dc_2^*}{dc_2} = 0 \tag{39}$$

Note that $g_1/g_1^* = \frac{dC}{dc_1}\frac{dc_1^*}{dC^*} = \frac{dC}{dC^*}\frac{dc_1^*}{dc_1} = -\frac{dC}{dC^*}$, therefore both of the above conditions imply (37), as in the FTA case. □

D Small-Open Economy

To further emphasize the importance of size in goods and financial markets, we next analyse the small open economy (SOE) limit of the model above. We follow CLW and define:

$$C^* = \frac{{c_1^*}^{\frac{1}{N}}{c_2^*}^{1-\frac{1}{N}}}{N-1} \tag{40}$$

where N is the number of countries. For simplicity, we consider:

$$C = c_1^{\frac{1}{2}}c_2^{\frac{1}{2}} \tag{41}$$

The market clearing equations equations are given by:

$$c_1 + c_1^* = y_1$$
$$c_2 + c_2^* = y_2 + (N-1)\ y_2^*$$

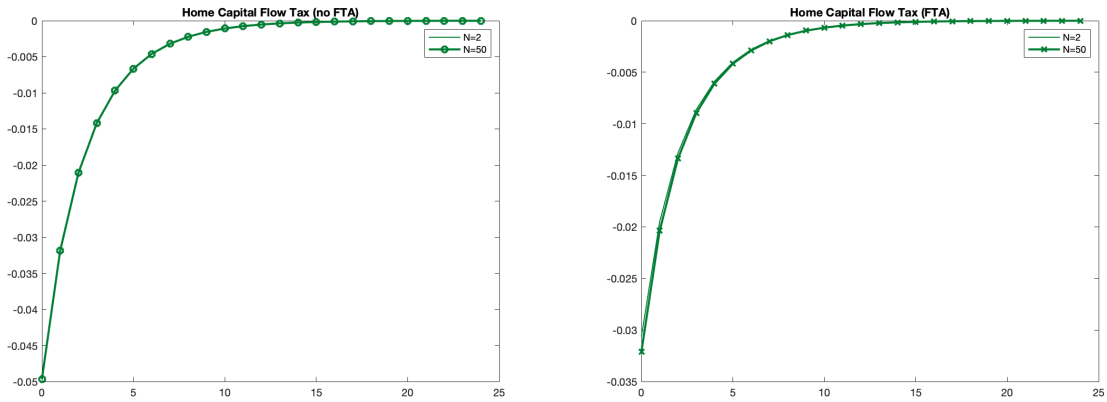


Figure 12: Time Profile of Optimal Taxes as the Home Endowment of Good 1 Rises in Scenario 1

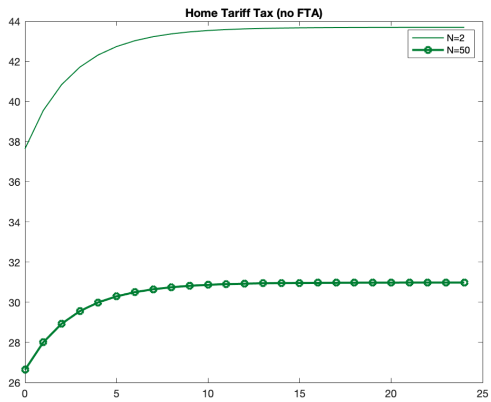


Figure 13: Time Profile of Optimal Tariffs as the Home Endowment of Good 1 Rises in Scenario 1

In the limit $N \to \infty$, the home country becomes a SOE. It follows that $C_t^* \to c_{2_t}^* = Y_{2,t}$ resulting in $\frac{dC^*}{dC} \to 0$.[26] The Home (SOE) planner maximizes utility subject to:

$$\sum_t (N-1)\ u'(C_t^*) \nabla g_t^* \cdot [\mathbf{c}_t - \mathbf{y}_t] \tag{42}$$

with the $(N-1)$ appearing because C^* is defined as per-country aggregate consumption.

[26] Moreover, as before, $\frac{dC_t^*}{dC_t} = -\frac{1}{Q_t} \to 0$ as $Q_t \to \infty$.