Inflation and Unemployment in General Equilibrium*

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Abstract

When labor is indivisible, there exist efficient outcomes with some agents randomly unemployed (Rogerson 1988). We integrate this idea into the modern theory of monetary exchange, where some trade occurs in centralized markets and some in decentralized markets (as in Lagos and Wright 2006). This delivers a general equilibrium model of unemployment and money, with explicit microeconomic foundations. We show the implied relation between inflation and unemployment can be positive or negative, depending on simple preference conditions. Our Phillips Curve provides a long-run, exploitable, trade off for monetary policy; it turns out, however, that the optimal policy is the Friedman rule.

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1 Introduction

The following has been well known at least since the work of Rogerson (1988): efficient allocations in economies with indivisible labor generally have some agents, chosen at random, unemployed, while others are employed, even if they are ex ante identical; and these allocations can be supported as competitive equilibria where agents trade lotteries. As we discuss in detail below, another interesting feature of these economies emphasized in Rocheteau et al. (2006) is that agents act as if they have quasi-linear utility. It turns out that one can use this result to construct a fairly general yet very tractable model of monetary exchange, using search theory, where as in Lagos and Wright (2005) some trades occur in centralized markets and some in decentralized markets.

To understand this, note that what makes Lagos-Wright tractable is the assumption of quasi-linear utility, which implies agents exiting the centralized market all hold the same amount of money, regardless of their histories (assuming interior solutions). Thus, if one is willing to embrace quasi-linear utility, one can avoid having to track the distribution of money in the decentralized market as a state variable.[1] The observations in Rocheteau et al. (2006) allow one to dispense with quasi-linearity: as long as we have indivisible labor – or for that matter, any indivisible commodity – identical results concerning the money distribution hold for *any* utility function

[1] For monetary models that are much less tractable because one has to keep track of the relevant distribution, see Green and Zhou (1998), Zhou (1999), Camera and Corbae (1999), Zhu (2003,2005) and Molico (2006). Earlier search-based models, such as Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991), Shi (1995), or Trejos and Wright (1995), were also simple, but only because they avoided the issue by assuming agents could only hold $m \in \{0, 1\}$ units of money. An alternative approach that uses large families to achieve tractability is provided by Shi (1997). See also Faig (2005).

(again assuming interior solutions). However, there are some advantages to using indivisible labor instead of quasi-linear utility as a building block for monetary theory, including the fact that it generates unemployment.

In this paper we take seriously the implications of indivisible-labor models with money for the relationship between inflation and unemployment. In other words, we study the Phillips curve in general equilibrium. This seems to be a natural exercise. In addition to Rogerson (1988), many well-known papers adopt the indivisible-labor model, including Hansen (1985), Cooley and Hansen (1989), Christiano and Eichenbaum (1992), Kydland (1994), Lungqvist and Sargent (2006), and Prescott, Rogerson and Wallenius (2006). These papers analyze macroeconomic issues either without money or with money added in some *ad hoc* way (e.g. via cash-in-advance constraints). We want to study the relation between unemployment and inflation in a model with microfoundations – i.e. with relatively explicit descriptions of the frictions that make money essential.[2]

The main goal is to derive precise analytic results showing how the relation between inflation and unemployment depends on primitives. We prove that the Phillips curve can have a positive or negative slope depending on the utility function in a simple and natural way. The intuition is straightforward. To consider one version of our results, suppose the economy has two sectors, one of which uses cash relatively intensively. Inflation is obviously a tax on activity in the cash-intensive sector. If goods traded in that sector are substitutes for those traded in the other sector, inflation reduces consump-

[2] An alternative approach, taken up in Berentsen, Menzio and Wright (2007), is to integrate unemployment into monetary theory using the search-based model of Mortensen and Pissardies (1994), instead of the indivisible labor model. As we discuss in the conclusion, there are advantages to each approach.

tion of the former and increases consumption of the latter. If those latter goods are relatively labor intensive, employment must increase in equilibrium. Hence, when the goods in the two sectors are substitutes, inflation reduces unemployment. Symmetrically, when the goods are complements inflation increases unemployment.

Our inflation-unemployment trade-off is based on rudimentary public finance considerations, and does not depend on any complicated features like sticky wages or prices, irrational expectations, imperfect information, etc. Thus we conclude that one does not have to work very hard to generate an interesting relation between inflation and unemployment. Now, in this paper, we do not take a stand on what the relation is in the data – that is an entirely different project. Here we focus on providing a simple characterization in theory. Also, note that our trade-off is a long-run trade-off in the sense that it does not depend on features like stickiness or imperfect information that are likely to be important only in the short run. And it is fully exploitable by monetary policy: under conditions we make precise, it is *feasible* to permanently reduce unemployment by increasing inflation, as Keynesians (used to?) think. We prove, however, that it is *optimal* to reduce inflation to a minimum, as Friedman always said.[3]

[3] A referee suggested that our results are reminiscent of Stockman (1981), who shows that inflation reduces captial in the long run when there is a cash-in-advance constraint on investment (as opposed to a cash-in-advance constraint on consumption only). Indeed, we acknowledge that one could derive results very similar to those in some of our propositions in indivisible-labor models where money is introduced in a reduced-form fashion by assuming some goods are subject to a cash-in-advance constraint and some goods are not, as long as one allows general preferences so that one can compare the cases where these goods are complements and when they are substitutes, and allows different labor intensities in production. We see no reason to take such a short cut, however, since it is no harder and it generates additional insights when one derives the role of money from first principles. But for those who are wed to reduced-form models, we empahsize that our main economic results also hold in that context.

2 Basic Assumptions

Time is discrete. There is a $[0,1]$ continuum of agents who live forever. There are two type of markets in which these agents interact. One is a frictionless centralized market, or CM. The other is a decentralized market, or DM, with two main frictions that together make money essential: a double-coincidence problem, as detailed below, and anonymity, which precludes private credit arrangements.[4] We are interested in equilibria where money is valued – i.e. the price of M is positive in the CM at every date – and choose dollars to be the unit of account. The stock of money evolves according to $\hat{M} = (1+\gamma)M$, where $\hat{z}$ indicates the value of any variable z next period. New money is injected (or withdrawn if $\gamma < 0$) via lump sum transfers (or taxes) in the CM, to neutralize the fiscal effects of monetary policy.[5]

It is easy to allow a general vector of consumption goods $\mathbf{x} \in \mathbb{R}^J_+$ and endowment $\mathbf{e} \in \mathbb{R}^J_+$ in the CM (see our working paper Rocheteau et al. 2007), but to reduce notation we assume a single CM good x and $e = 0$. This good is produced by firms using labor h. For an individual, labor is indivisible: $h \in \{0,1\}$. Hence, as is standard, agents trade randomized consumption bundles, or *lotteries*, in the CM. In the DM there is a different good, q, that is not produced, but agents have an endowment $\bar{q}$. Utility in an interval encompassing one CM and DM is $v^j(q,x,h)$, where j is a preference

[4] See Kocherlakota (1998), Wallace (2001), Corbae, Temzilides and Wright (2003), or Aliprantis, Camera and Puzello (2007) for formal discussions of essentiality and the role of anonymity. Aliprantis et al. is especially relevant since they explicitly consider models with both centralized and decentralized meetings.

[5] As Todd Keister pointed out to us, under the alternative assumption that increasing M means increasing government spending, in this model, an increase in γ always reduces unemployment – this is the wealth effect of fiscal policy on employment emphasized by Christiano and Eichenbaum (1992). While this is interesting, and potentially quantitatively relevant, we want to focus on the effects of monetary policy on the return to holding currency, rather than the pure wealth effect, so we hold government spending constant.

shock realized at the start of the DM, after (x, h) but before q is chosen. This timing, and in particular the assumption that the preference shock is revealed after (x, h) but before q is chosen, is not especially important, but we like the interpretation that the CM closes before the DM opens.

The related monetary literature assumes utility is separable between (x, h) and q, but for reasons that will become clear we do not want to restrict attention to this case. Indeed, papers following Lagos and Wright (2005) assume utility is linear in either x or h, but as will become clear we do not need to restrict attention to this case. Although it would be straightforward to allow general preference or endowment shocks, it suffices here to assume the following: with probability σ the utility function is υ^H, and with probability $1-\sigma$ it is υ^L, where $\partial \upsilon^H(q, x, h)/\partial q > \partial \upsilon^L(q, x, h)/\partial q \; \forall (q, x, h)$, which generates gains from trade similar to e.g. Berentsen and Rocheteau (2003). The key point for our purposes is that there is no production in the DM – it is a pure exchange market – and so employment is unambiguously given by hours worked in the CM.

Trade in the DM is bilateral, and so we have to say how people meet. Although one could consider a more general matching technology (see Rocheteau et al. 2007), here we set $\sigma = 1/2$ and assume every agent that draws υ^H is matched with one that draws υ^L. We call the former a buyer and the latter a seller, since there is a deal to be done where the agent with υ^L transfers some q to the one with υ^H in exchange for some cash. Generally, the frictions in the DM imply an essential role for *some* medium of exchange. We assume that the good x cannot be carried into the DM, and there is no other storable object, so that this role must be played by money (in other words, this paper is *not* about the coexistence of money and other

assets).

We impose standard curvature conditions on utility to guarantee consumption of x is strictly positive. With indivisible labor, one obviously cannot do something similar for h, and interiority of (the probability of) employment is an issue to which we return below. We assume agents discount between the DM and next CM at rate $\beta \in (0,1)$, but not between the CM and DM. Let $W(m)$ denote the CM value function, which depends only on money balances, since in all other respects agents are identical in this market. Let $V(m,x,h)$ denote the DM value function, which, in addition to m, also depends on (x,h) since these are given when one enters this market. This completes our description of the basic model. In the next four sections we analyze the CM, we analyze the DM, we put them together to define equilibrium, and we discuss the implications for the relation between inflation and unemployment.

3 The CM

As we said, going back to Rogerson (1988), in models with indivisible labor agents trade lotteries. Thus, in the CM, given m, an agent chooses $(\ell, x_1, x_0, \widetilde{m}_1, \widetilde{m}_0)$, where ℓ is the probability of employment (i.e. of $h=1$), while x_1 and $\widetilde{m}_1$ are consumption and money if employed and x_0 and $\widetilde{m}_0$ are consumption and money if unemployed. The problem is

$$W(m) = \max_{(\ell,x_1,x_0,\widetilde{m}_1,\widetilde{m}_0)} \{\ell V(\widetilde{m}_1, x_1, 1) + (1-\ell)V(\widetilde{m}_0, x_0, 0)\} \qquad (1)$$

$$\text{s.t. } \ell(px_1 + \widetilde{m}_1) + (1-\ell)(px_0 + \widetilde{m}_0) \leq w\ell + m + \gamma M + \Delta,$$

where p is the price of x, w the wage, γM the lump sum money transfer, and Δ dividend income, all measured in dollars. As is standard, agents

effectively get paid for selling a probability of working ℓ. What is slightly non-standard here is that they do not derive utility directly in this market; instead they take $(\widetilde{m}, x, h)$ to the DM, where other events occur.

A few additional comments are in order concerning (1). First, as is well known in related models, an individual's choice will generally be contingent on his employment status h, but if V is separable between x and h then $x_0 = x_1$, and if V is separable between $\widetilde{m}$ and h then $\widetilde{m}_1 = \widetilde{m}_0$.[6] Second, Rocheteau et al. (2006) actually derive problem (1) from a model with standard Arrow-Debreu markets, and no lotteries, where agents trade state-contingent commodity bundles $[x(s), h(s), \widetilde{m}(s)]$ and s represents a sunspot; for simplicity we start directly with lotteries.[7] Finally, although (1) generally does not have a quasi-concave objective function, Rocheteau et al. (2006) show there is a unique solution and the second-order conditions hold.

Although this is very easy to generalize, to reduce the notation we assume an aggregate production function that converts h into x one-for-one, so that in any equilibrium $w = p$ and $\Delta = 0$. Hence, the Lagrangian for (1) can be written

$$\begin{aligned} W(m) &= \ell V(\widetilde{m}_1, x_1, 1) + (1-\ell) V(\widetilde{m}_0, x_0, 0) \\ &\quad + \lambda \left[\ell - \ell x_1 - (1-\ell) x_0 + \frac{m + \gamma M - \ell \widetilde{m}_1 - (1-\ell)\widetilde{m}_0}{p} \right] \end{aligned}$$

where λ is a multiplier. Assuming an interior solution for $\ell \in (0,1)$ for now

[6] Of course, $V(\cdot)$ is an endogenous object, and whether it is separable depends on the underlying utility function $v^j(q, x, h)$. One advantage of deriving the role for money explicitly, instead of just sticking it in the utility function, say, is that it imposes some discipline: one cannot simply *assume* e.g. that $\widetilde{m}$ and h enter separably. In any case, it will be very clear below if $x_1 = x_0$ or $\widetilde{m}_1 = \widetilde{m}_0$.

[7] That result is based on Shell and Wright (1993), where it is shown how to support efficient allocations in nonconvex economies as sunspot equilibria instead of lottery equilibria. Sunspots have advantages over lotteries, in general, but in some contexts, including this model, they are equivalent (see Garratt, Keister and Shell 2004).

(but see below), first-order conditions are

$$x_h \quad : \quad V_x(\widetilde{m}_h, x_h, h) = \lambda, \; h = 0, 1 \tag{2}$$

$$\widetilde{m}_h \quad : \quad V_m(\widetilde{m}_h, x_h, h) = \lambda / p, \; h = 0, 1 \tag{3}$$

$$\ell \quad : \quad V(\widetilde{m}_1, x_1, 1) - V(\widetilde{m}_0, x_0, 0) = \lambda \left(x_1 - x_0 - 1 + \frac{\widetilde{m}_1 - \widetilde{m}_0}{p} \right) \tag{4}$$

$$\lambda \quad : \quad \ell - \ell x_1 - (1 - \ell) x_0 + \frac{m + \gamma M - \ell \widetilde{m}_1 - (1 - \ell) \widetilde{m}_0}{p} = 0. \tag{5}$$

Since $V(\widetilde{m}_h, x_h, h)$ does not depend on m, (2)-(4) constitute 5 equations that can be solved for $(x_1, x_0, \widetilde{m}_1, \widetilde{m}_0, \lambda)$, independent of ℓ and m, under weak regularity conditions.[8] Once we have $(x_1, x_0, \widetilde{m}_1, \widetilde{m}_0, \lambda)$, (5) yields ℓ as the following function of m:

$$\ell = \ell(m) = \frac{p x_0 + \widetilde{m}_0 - \gamma M - m}{p(1 + x_0 - x_1) + \widetilde{m}_0 - \widetilde{m}_1}. \tag{6}$$

Individuals with more money supply less labor in the CM – i.e. they work with a lower probability – which is how everyone can afford the same $(x_1, x_0, \widetilde{m}_1, \widetilde{m}_0, \lambda)$. In terms of monetary economics, a key part of the result is that the choice of $\widetilde{m}_h$ may depend on employment status but not on m, and hence all agents take the same money holdings out of the CM conditional on h.

This is similar to the basic result in the Lagos-Wright model, where all agents take the same money holdings out of the CM, and hence we get a degenerate distribution of $\widetilde{m}$ in the DM. Here we get at most a two-point distribution, since in general the employed and unemployed do not have the

[8] See Rocheteau et al. (2006) for details, but basically the regularity condition is that we need to rule out $p(x_1 - x_0 - 1) = \widetilde{m}_0 - \widetilde{m}_1$, or equivalently $V(\widetilde{m}_1, x_1, 1) = V(\widetilde{m}_0, x_0, 0)$, which implies a singularity in (2)-(4). In the indivisible-labor literature, this case is known to occur only for one special utilty function (see e.g. Cooper 1987 or Rogerson and Wright 1988). Actually, even in this case, our results go through, but we need a different argument. Again, see Rocheteau et al. (2006).

same $\widetilde{m}$, although as we said above they do when V is separable between $\widetilde{m}$ and h. In any case, a two-point distribution is not hard to handle; the important part is to eliminate history dependence (the dependence of $\widetilde{m}$ on m). We empathize that the result in the Lagos-Wright model only holds with quasi-linear utility, while here it holds for *any* utility function. Another result that carries over from quasi-linear models is that $W(m)$ is linear: $W'(m) = \lambda/p$, by the envelope theorem, where we already established that λ is independent of m. One might say agents in the indivisible-labor model act *as if* they have quasi-linear utility. Summarizing:

Lemma 1 *Assuming $\ell \in (0,1)$ for all agents in the CM the choice of $\widetilde{m}_h$ and $W'(m) = \lambda/p$ are independent of m.*

4 The DM

Consider a meeting where the buyer has m^b and the seller m^s dollars, for arbitrary values of m^b and m^s. Generally, they trade d dollars for q units of the good, subject to $d \leq m^b$ and $q \leq \bar{q}$, since neither agent can turn over more than he has. There are several ways to determine the terms of trade (q, d): Lagos and Wright (2005) use generalized Nash bargaining; Aruoba, Rocheteau and Waller (2007) consider several alternative bargaining solutions; Rocheteau and Wright (2005) consider price taking (as in the Lucas-Prescott 1974 labor-market search model) and price posting (as in the directed search model of Moen 1996 and Shimer 1995); and Galenianos and Kircher (2006) and Dutu, Julien and King (2007) use auctions (in versions of the model that allow for some multilateral matches, which we could easily accommodate). Although it would clearly be interesting to consider

various alternatives, in this paper we stick to bargaining, and indeed to take-it-or-leave-it offers by the buyer.

The reason is the following: There is a complication here, compared to many models, because there are, in principle, several types of meetings that can occur between buyers and sellers depending on their employment status in the CM – an employed buyer could meet an employed seller, an employed buyer could meet an unemployed seller, etc. But if we adopt take-it-or-leave-it offers by the buyer, and if we also assume that sellers' preferences are separable, say $v^L(q,x,h) = F(x,h) + G(q)$, then the terms of trade will not depend on anything in the meeting except the money holdings of the buyer m^b. Although it will be important below to allow buyers' preferences to be non-separable between (x,h) and q, we can still get interesting results when sellers' preferences are separable; therefore we focus on this case to facilitate the presentation.[9]

Given the buyer makes a take-it-or-leave-it offer, one can easily show that in any equilibrium the buyer offers all his money $d = m^b$, and asks for the q that makes the seller indifferent between trading and not trading

$$G(\bar{q} - q) + \beta \hat{W}(m^s + m^b) = G(\bar{q}) + \beta \hat{W}(m^s),$$

where $\hat{W}(m)$ is the CM value function next period.[10] By Lemma 1, $\hat{W}(m^s +$

[9] In Rocheteau et al. (2007), we present the case where the buyer has general bargaining power θ, but since we could only prove the main results (Propositions 2 and 3) for $\theta = 1$, on the suggestion of a referee we restrict attention here to this special case. This is somewhat unfortunate, however, since $\theta = 1$ does preclude many interesting extensions – e.g. it rules out a version of the model where sellers have to pay a fixed entry costy to participate in the DM.

[10] See Lagos and Wright (2005) for details, but here is the idea. It is easy to check that a buyer spends all his money iff m^b is below some threshold m^* and that $m^b \leq m^*$ in equilibrium. Also, one can show that q is increasing in m^b and $m^b = m^*$ implies $q = q^*$ where $u'(q^*) = c'(q^*)$. This will imply below that $q \leq q^*$ in equilibrium, with strict equality iff the nominal interest rate is $i = 0$.

$m^b) - \hat{W}(m^s) = m^b \hat{\lambda}/\hat{p}$, and the previous expression reduces to $G(\bar{q}) - G(\bar{q} - q) = \beta m^b \hat{\lambda}/\hat{p}$. In general, some buyers were employed while some were unemployed in the CM, and the former have $\widetilde{m}_1$ and trade for $q_1 = q(\widetilde{m}_1)$ while the latter have $\widetilde{m}_0$ and trade for $q_0 = q(\widetilde{m}_0)$ in the DM. But the deal is independent of the value of (x_h, h) for either the buyer or the seller as well as m^s. Summarizing:

Lemma 2 *The DM bargaining solution is $d = m^b$ and $q = q(m^b)$, where the function $q(\cdot)$ is the solution to*

$$\beta m^b \hat{\lambda}/\hat{p} = G(\bar{q}) - G(\bar{q} - q). \tag{7}$$

Now, for an agent in the DM with $(\widetilde{m}_h, x_h, h)$, when every other agent has some possibly random amount of money μ,

$$\begin{aligned} V(\widetilde{m}_h, x_h, h) &= \frac{1}{2}\left\{ \upsilon^H \left[\bar{q} + q\left(\widetilde{m}_h\right), x_h, h \right] + \beta \hat{W}\left(0\right) \right\} \\ &\quad + \frac{1}{2} \mathbb{E}_\mu \left\{ \upsilon^L \left[\bar{q} - q\left(\mu\right), x_h, h \right] + \beta \hat{W}(\widetilde{m} + \mu) \right\}, \end{aligned}$$

since with probability 1/2 he is a buyer and with probability 1/2 he is a seller. Inserting the separable form of $\upsilon^L(\cdot)$,

$$\begin{aligned} V(\widetilde{m}_h, x_h, h) &= \frac{1}{2}\left\{ \upsilon^H \left[\bar{q} + q\left(\widetilde{m}_h\right), x_h, h \right] + \beta \hat{W}\left(0\right) \right\} \\ &\quad + \frac{1}{2} \mathbb{E}_\mu \left\{ F(x_h, h) + G[\bar{q} - q\left(\mu\right)] + \beta \hat{W}(\widetilde{m} + \mu) \right\}. \end{aligned} \tag{8}$$

Differentiating, and inserting $\hat{W}'(\cdot)$ and $q'(\cdot)$ from Lemmas 1 and 2, we arrive at the envelope conditions

$$V_m(\widetilde{m}_h, x_h, h) = \frac{\beta}{2} \frac{\hat{\lambda}}{\hat{p}} \left\{ \frac{\upsilon_q^H \left[\bar{q} + q(\widetilde{m}_h), x_h, h \right]}{G'\left[\bar{q} - q(\widetilde{m}_h) \right]} + 1 \right\} \tag{9}$$

$$V_x(\widetilde{m}_h, x_h, h) = \frac{1}{2} \upsilon_x^H \left[\bar{q} + q(\widetilde{m}_h), x_h, h \right] + \frac{1}{2} F_x(x_h, h). \tag{10}$$

5 Equilibrium

We begin by collecting some results. First, insert the expressions for V and its derivatives in (8)-(10) plus the bargaining solution (7) from the DM into the first-order conditions (2)-(4) from the CM and simplify to get

$$\lambda = \frac{1}{2}\upsilon_x^H\left[\bar{q}+q(\widetilde{m}_h),x_h,h\right]+\frac{1}{2}F_x(x_h,h),\ h=0,1 \tag{11}$$

$$\frac{\lambda}{p} = \frac{\beta}{2}\frac{\hat{\lambda}}{\hat{p}}\left\{\frac{\upsilon_q^H\left[\bar{q}+q(\widetilde{m}_h),x_h,h)\right]}{G'\left[\bar{q}-q(\widetilde{m}_h)\right]}+1\right\},\ h=0,1 \tag{12}$$

$$\begin{aligned} 0 = {} & \left(\frac{\lambda\hat{p}}{\hat{\lambda}p\beta}-\frac{1}{2}\right)\left\{G\left[\bar{q}-q(\widetilde{m}_1)\right]-G\left[\bar{q}-q(\widetilde{m}_0)\right]\right\}+\lambda\left(1-x_1+x_0\right) \\ & +\frac{1}{2}\left\{\upsilon^H\left[\bar{q}+q(\widetilde{m}_1),x_1,1\right]-\upsilon^H\left[\bar{q}+q(\widetilde{m}_0),x_0,0\right]+F(x_1,1)-F(x_0,0)\right\} \end{aligned} \tag{13}$$

Next, integrate (6) over agents with different m to get

$$p\left[\bar{\ell}-\bar{\ell}x_1-(1-\bar{\ell})x_0\right]=\bar{\ell}\widetilde{m}_1+(1-\bar{\ell})\widetilde{m}_0-\hat{M},$$

which says the money market clears iff the goods market clears (Walras' Law). Goods market clearing implies

$$\bar{\ell}=\frac{x_0}{1+x_0-x_1}. \tag{14}$$

Also, by Lemma 2

$$\bar{\ell}\widetilde{m}_1+(1-\bar{\ell})\widetilde{m}_0=\left\{\bar{\ell}\left[G(\bar{q})-G(\bar{q}-q_1)\right]+(1-\bar{\ell})\left[G(\bar{q})-G(\bar{q}-q_0)\right]\right\}\frac{\hat{p}}{\beta\hat{\lambda}},$$

and money market clearing implies

$$\hat{p}=\frac{\beta\hat{\lambda}\hat{M}}{\bar{\ell}\left[G(\bar{q})-G(\bar{q}-q_1)\right]+(1-\bar{\ell})\left[G(\bar{q})-G(\bar{q}-q_0)\right]}. \tag{15}$$

Although one can proceed more generally, we focus on steady states where all real variables are constant.[11] Then (15) pins down inflation by

[11] Dynamics are discussed briefly in a footnote below. See e.g. Lagos and Wright (2003) for a general treatment of dynamics in these kinds of models.

$\hat{p}/p = \hat{M}/M = 1+\gamma$. Also, we can use the Fisher equation $1+i = \hat{p}/p\beta = (1+\gamma)/\beta$ to define the nominal interest rate i.[12] Then (11)-(13) become

$$\lambda = \lambda = \frac{1}{2}\upsilon_x^H(\bar{q}+q_h, x_h, h) + \frac{1}{2}F_x(x_h, h),\ h = 0,1 \tag{16}$$

$$i = \frac{1}{2}\left[\frac{\upsilon_q^H(\bar{q}+q_h, x_h, h)}{G'(\bar{q}-q_h)} - 1\right],\ h = 0,1 \tag{17}$$

$$\begin{aligned} 0 = {} & \left(\frac{1}{2}+i\right)\left[G(\bar{q}-q_1) - G(\bar{q}-q_0)\right] + \lambda\left(1-x_1+x_0\right) \\ & + \frac{1}{2}\left[\upsilon^H\left(\bar{q}+q_1, x_1, 1\right) - \upsilon^H\left(\bar{q}+q_0, x_0, 0\right) + F(x_1,1) - F(x_0,0)\right], \end{aligned} \tag{18}$$

where we write q_h for $q(\widetilde{m}_h)$. System (16)-(18) constitutes 5 equations in $(x_1, x_0, q_1, q_0, \lambda)$. Given this, aggregate employment is given by (14), the price level by (15), and individual money holdings by $\widetilde{m}_h = p(1+\gamma)\left[G(\bar{q}) - G(\bar{q}-q_h)\right]/\beta\lambda$.

A solution to the above set of equations defines a steady state equilibrium, subject to one caveat: we need to discuss the maintained assumption $0 < \ell(m) < 1$ for all m in the support of the equilibrium distribution of money holdings across agents entering the CM, upon which much of the above analysis is based.[13] We now discuss conditions under which the maintained assumption is valid. First, recall that $\ell(m)$ is given by (6), the denominator of which, $p(1+x_0-x_1)+\widetilde{m}_0-\widetilde{m}_1$, can be positive or negative. We concentrate on the former case, in which we have $\ell'(m) < 0$, and leave

[12] The Fisher equation can be interpretted as a no-arbitrage condition for pricing a hypothetical nominal asset purchased in one CM and paying off in the next CM, which by assumption cannot be brought to (traded in) the DM; or one can interpret it merely as a piece of notation defining i in terms of β and the exogenous money growth rate γ.

[13] It is not that there is anything wrong in principle with equilibria with $\ell(m) = 0$ or $\ell(m) = 1$ for some m, but in practice the algebra becomes complicated. The main reason is that, when some agents hit corner solutions, we cannot guarantee they will all bring the same amount of money (coningent on employment status) to the DM. One then has to keep track of the DM money distribution as a state variable, which makes analytic results difficult. However, one can use numerical methods, as e.g. Molico (2006) or Chiu and Molico (2007) do in closely related models.

the latter as an exercise.[14] After using money market clearing to eliminate M and goods market clearing to eliminate $\bar{\ell}$, we have

$$\ell(m) = \frac{(1+\gamma)(1-x_1+x_0)px_0+(1-x_1+x_0+\gamma x_0)\widetilde{m}_0-\gamma x_0\widetilde{m}_1-(1-x_1+x_0)(1+\gamma)m}{(1+\gamma)(1-x_1+x_0)[p(1-x_1+x_0)+\widetilde{m}_0-\widetilde{m}_1]}. \quad (19)$$

We use this to check whether $0 < \ell(m) < 1$ in equilibrium.

Note that all agents who were buyers in the previous DM enter the CM with $m = 0$, while those who were sellers enter with the money they brought themselves plus what they acquired from sales. Letting $\bar{m} = \max\{\widetilde{m}_0, \widetilde{m}_1\}$, the most an agent could have entering the CM is therefore $2\bar{m}$. Since we are considering the case where $\ell'(m) < 0$, we have $0 < \ell(m) < 1$ for all m in the support of the equilibrium distribution iff $\ell(0) < 1$ and $\ell(2\bar{m}) > 0$. By (19) these inequalities are equivalent to:

$$\begin{aligned} x_1 &< 1 - \frac{(1 - x_1 + x_0 + \gamma - \gamma x_1)\widetilde{m}_1 - \gamma(1 - x_1)\widetilde{m}_0}{p(1+\gamma)(1 - x_1 + x_0)} \\ x_0 &> \frac{\gamma x_0\widetilde{m}_1 - (1 - x_1 + x_0 + \gamma x_0)\widetilde{m}_0 - 2(1 - x_1 + x_0)\bar{m}}{p(1+\gamma)(1 - x_1 + x_0)} \end{aligned}$$

which can be checked for any solution to the set of equations defining equilibrium. In some cases this is really quite easy. For instance, if utility is separable between q and (x, h), so that $\widetilde{m}_1 = \widetilde{m}_0 = \bar{m}$ are all equal to $M(1+\gamma)$, these inequalities reduce rather dramatically to

$$x_1 < 1 - \frac{M}{p} \text{ and } x_0 > \frac{M}{p}.$$

One can also express this in terms of consumption goods by inserting $M/p = p(1+\gamma)\left[G(\bar{q}) - G(\bar{q} - q_h)\right]/\beta\lambda$. If e.g. utility is also separable

[14] The case where it is 0 is ruled out by the regularity condition mentioned in fn. 8. Note that the denominator is positive iff the unemployed are better off than the employed in the CM, which is actually the more common if not the only possible case – e.g. if utility is separable between h and (x, q), then the employed and unemployed get the same consumption, while the latter enjoy leisure and hence are clearly better off.

between x and h then things are especially neat, since then x and λ are independent of q, and since $q \leq q^*$ in any equilibrium, where q^* is the first best quantity, we simply need to choose preferences so that q^* is small. Intuitively, if the value of money is too big, either agents with $m = 0$ would have to supply $\ell(m) > 1$ to get back up to the equilibrium $\widetilde{m} = M(1+\gamma)$, or those with $m = 2M$ would have to supply $\ell(m) < 0$ to get back down to $M(1+\gamma)$; hence we need q to be not too big. In numerical calculations, it was not hard to choose parameters to guarantee $0 < \ell(m) < 1$ for all relevant m, or to choose other parameters where the constraint $\ell(m) \in [0,1]$ is binding. In what follows, we do not dwell on this, and simply take for granted that conditions hold so as to guarantee $0 < \ell(m) < 1$ in equilibrium.

6 The Phillips Curve

We now study the relation between unemployment and monetary policy, where policy here can be described in terms of either inflation γ or the nominal interest i, by virtue of $(1+i) = (1+\gamma)/\beta$. We separate the analysis into three cases, depending on buyers' preferences. In the first case, as in most of the related literature, utility is separable between the CM and DM allocations (x, h) and q; in the second case q interacts with x; and in the final case q interacts with h. Also, mainly to reduce the notation, for this exercise we assume sellers' utility is separable in all three arguments, $v^L(q, x, h) = F(x) + G(q) + H(h)$. Summarizing, the cases are:

1. Case 1: $v^H(q, x, h) = f(x, h) + g(q)$
2. Case 2: $v^H(q, x, h) = f(q, x) + g(h)$
3. Case 3: $v^H(q, x, h) = f(q, h) + g(x)$

In the first case, (17) reduces to

$$i = \frac{1}{2}\left[\frac{g'(\bar{q}+q_h)}{G'(\bar{q}-q_h)} - 1\right], \; h = 0, 1. \tag{20}$$

Since (20) has a unique solution for q_h, we have $q_1 = q_0 = q$. Moreover, we can solve for q independently of the rest of the equilibrium. Then (16-(18) reduce to

$$\lambda = \frac{1}{2}\left[f_x(x_h, h) + F'(x_h)\right], \; h = 0, 1 \tag{21}$$

$$0 = \frac{1}{2}\left[f(x_1, 1) + F(x_1) - f(x_0, 0) - F(x_0)\right] - \lambda\left(x_1 - x_0 - 1\right). \tag{22}$$

These 3 equations can be solved for (x_1, x_0, λ), from which we get $\bar{\ell} = x_0/\left(1 + x_0 - x_1\right)$ and the rest of the equilibrium. Notice that policy i impacts on q, with $\partial q/\partial i < 0$, but not at all on (x_1, x_0, λ) or $\bar{\ell}$. One might recognize this as a version of the neoclassical dichotomy, which in the current context has the implication that the Phillips curve is vertical.[15]

Proposition 1 *In case 1,* $\partial q/\partial i < 0$ *and* $\partial \bar{\ell}/\partial i = 0$.

In case 2 things are quite different. First, (16) and (17) now become

$$\lambda = \frac{1}{2}\left[f_x(\bar{q}+q_h, x_h) + F'(x_h)\right], \; h = 0, 1$$

$$i = \frac{1}{2}\left[\frac{f_q(\bar{q}+q_h, x_h)}{G'(\bar{q}-q_h)} - 1\right], \; h = 0, 1.$$

[15] As Sargent (1979) puts it, "A macroeconomic model is said to dichotomize if a subset of equations can determine the values of all real variables with the level of the money supply playing no role in determining the equilibrium value of any real variable. Given the equilibrium values of the real variables, the level of the money supply helps determine the equilibrium values of all nominal variables that are endogenous but cannot influence any real variable. In a system that dichotomizes the equilibrium values of all real variables are independent of the absolute price level." The model here does not dichotomize into real and nominal parts, where money can only affect the latter, since q is a real variable; it instead dichotomizes under this special specification into CM and DM parts, just like the baseline Lagos-Wright model; see Aruoba and Wright (2003). Aruoba, Waller and Wright (2005) break the dichotomy by introducing capital that is produced in the CM and used to make DM goods. The plan here is to break the dichotomy by considering non-separable utility between CM and DM goods.

These conditions imply $x_1 = x_0 = x$ and $q_1 = q_0 = q$. Then (18) pins down $\lambda = \bar{\lambda} \equiv \frac{1}{2}\left[g(0) - g(1) + H(0) - H(1)\right]$. Now we can summarize equilibrium conveniently as the solution (q, x) to

$$0 = f_x(\bar{q} + q, x) + F'(x) - 2\bar{\lambda} \tag{23}$$

$$0 = f_q(\bar{q} + q, x) - (2i + 1)\, G'(\bar{q} - q_h). \tag{24}$$

Therefore, $\partial q/\partial i = 2G'\left(f_{xx} + F'\right)/D < 0$ and $\partial x/\partial i = -2G' f_{xq}/D \backsimeq -f_{xq}$, where $a \backsimeq b$ means a and b are equal in sign and

$$D \equiv f_{xx} f_{qq} - f_{xq}^2 + F'' f_{qq} + \left(f_{xx} + F''\right)(2i + 1)G'' > 0.$$

Thus, q unambiguously falls with i, while effect on x depends on the cross derivative f_{xq}, and since $\bar{\ell} = x$ employment increases iff x increases.

Proposition 2 *In case 2, $\partial q/\partial i < 0$ and $\partial \bar{\ell}/\partial i > 0$ iff $f_{xq} < 0$.*

This is very intuitive. Inflation is a direct tax on DM activity, and hence reduces q. If $f_{xq} > 0$ (q and x are complements) then inflation also reduces x, and hence $\bar{\ell}$. But if $f_{xq} < 0$ (q and x are substitutes) then inflation increases x and $\bar{\ell}$. In the latter case, inflation causes people to substitute out of DM goods and into CM goods, increasing CM production and reducing unemployment. We thus get a Phillips curve under simple and natural conditions. Perhaps the most surprising part is that the results are so clean – why e.g. are there no ambiguous wealth and substitution effects? The reason is the same as the reason why the model is so tractable, in general: agents here act as if they have quasi-linear utility.

Before moving on, we mention that this model has a neat graphical representation. Condition (23) implies $x = X(q)$ and (24) implies $q = Q(x)$,

which have slopes in (x, q) space that depend on f_{xq}:

$$\frac{dq}{dx}_{|x=X(q)} = \frac{-(f_{xx} + F'')}{f_{xq}} \text{ and } \frac{dq}{dx}_{|q=Q(x)} = \frac{-f_{xq}}{(f_{xx} + G'')}.$$

See Figure 1. Standard assumptions can be imposed to imply these curves must cross, guaranteeing existence. A simple calculation (the same one that shows $D > 0$) shows the $X(q)$ curve must be steeper than the $Q(x)$ curve, implying uniqueness of monetary steady state.[16] An increase in i shifts the $Q(x)$ curve down, leading to a fall in q and increase or a decrease in x when we have $f_{xq} < 0$ or $f_{xq} > 0$. Hence, existence, uniqueness and comparative statics can all be described in one simple diagram.[17]

Something similar happens in case 3, so we merely sketch the analysis. First, (16) becomes

$$\lambda = \frac{1}{2}\left[g'(x_h) + F'(x_h)\right], \, h = 0, 1, \tag{25}$$

which implies $x_1 = x_0 = x$, but now (17) becomes

$$i = \frac{1}{2}\left[\frac{f_q(\bar{q} + q_h, h)}{G'(\bar{q} - q_h)} - 1\right], \, h = 0, 1, \tag{26}$$

which implies $q_0 \neq q_1$ in this case, with $q_1 > q_0$ iff $f_{qh} > 0$. Then (18) becomes

$$\begin{aligned} 0 \;\; = \;\; & \frac{1}{2}\left[f(\bar{q} + q_1, 1) - f(\bar{q} + q_0, 0) + H(1) - H(0)\right] \\ & - \left(\frac{1}{2} + i\right)\left[G(\bar{q} - q_1) - G(\bar{q} - q_0)\right] - \lambda. \end{aligned} \tag{27}$$

[16] There is of course also a non-monetary steady state, which is a value for x solving (23) with $q = 0$, but we are ignoring that here.

[17] One can also depict the necessary and sufficient conditions for $0 < \ell(m) < 1$, since in this case they reduce to $[G(\bar{q}) - G(\bar{q} - q)] < \beta\bar{\lambda}x < \beta\bar{\lambda} - [G(\bar{q}) - G(\bar{q} - q)]$, which we can easily plot in (x, q) space. One can also describe efficiency in the figure using Proposition 4 below. Finally, one use the figure to discuss dynamics using standard methods: it is not hard to show that if we peg the interest rate i, then the steady state is the only monetary equilibrium; but if we peg γ, then in addition there exist dynamic equilibria converges to the nonmonetary steady state over time along the $X(q)$ curve.

We leave as an exercise verification of $\partial q_h/\partial i < 0$ for $h = 0, 1$ and $\partial \bar{\ell}/\partial i \lesseqgtr -f_{qh}$. The important point is, once again, that employment increases or decreases with inflation under simple conditions on the cross derivative.

Proposition 3 *In case 3,* $\partial q_h/\partial i < 0$ *and* $\partial \bar{\ell}/\partial i > 0$ *iff* $f_{qh} < 0$.

This result is also very intuitive. As always, in the model, inflation reduces q. If $f_{qh} > 0$ (q and h are complements, or q and leisure substitutes) the reduction in q increases leisure and hence reduces employment. But if $f_{qh} < 0$ (q and leisure are complements) then, by reducing q, inflation also reduces leisure and hence decreases equilibrium unemployment. Again, we again get a Phillips curve under very simple and natural conditions. Whether or not one sees a downward-sloping, an upward-sloping, or a vertical curve in the data is another issue altogether, and not one we consider here. The point is simply that there is no problem in theory accounting for a relation between inflation and unemployment.

Our results are all based on rudimentary public finance considerations, and do not require anything "tricky" like signal extraction problems, nominal rigidities, and so on.[18] The other point to emphasize is that these considerations lead to a relation between inflation and employment that is stable and exploitable in the long run. Thus, given the right cross derivatives, our propositions indicate that policy makers *can* achieve a permanently lower rate of unemployment by printing money at a faster rate. However, we now argue that this is not a good idea.

[18] One might say that there is one "tricky" ingredient here. We set things up to get unambiguous results by assuming that the DM is a pure exchange market, and labor is not used to produce q. More generally, as long as x production is relatively labor intensive, compared to q, it will be possible to get similar results; the exact conditions will change, but the economic content will be similar.

To this end, consider the planner's problem:

$$\mathcal{W} = \max_{(\ell,x_1,x_0,q_1,q_0)} \{\frac{\ell}{2}\upsilon^H(\bar{q}+q_1,x_1,1) + \frac{1-\ell}{2}\upsilon^H(\bar{q}+q_0,x_0,0) + \frac{\ell}{2}\upsilon^L(\bar{q}-q_1,x_1,1) + \frac{1-\ell}{2}\upsilon^L(\bar{q}-q_0,x_0,0) + \beta\mathcal{W}\}$$
$$\text{s.t. } \ell x_1 + (1-\ell)x_0 \leq \ell$$

It is easy to check that, given our assumptions, the first-order conditions for this problem coincide with the equilibrium conditions iff $i = 0$. Hence, the efficient policy is the Friedman rule, regardless of whether the Phillips curve is upward-sloping, downward-sloping, or vertical. Of course, we set up the framework intentionally avoiding externalities, market incompleteness, and so on. If one is free to add features such as these, it should not be difficult to get equilibrium unemployment to be too high or too low, in which case the analysis might lead to different policy conclusions – although it is not so clear that inflation is the only or the best tax for addressing these issues. In any case, in the benchmark model we have the following:

Proposition 4 *The Friedman rule $i = 0$ achieves the solution to the planner's problem.*

7 Conclusion

In nonconvex economies, randomized allocations are desirable and can be supported as equilibria with lotteries. This not only generates unemployment, it is very convenient for monetary theory, because it provides an alternative to Lagos and Wright (2005) in the sense that it generates tractability without quasi-linearity. We used these ideas to construct a general equilibrium model of the relation between inflation and unemployment. We prove

that the Phillips curve slopes either up or down, depending on cross derivatives of the utility function. The idea is simple and plausible: inflation is a tax on economic activity that uses cash, and so if either labor-intensive goods are substitutes for this activity or leisure is complementary with this activity, inflation reduces unemployment. This does not mean we should use inflation to combat unemployment, since (without additional complications) the optimal policy is the Friedman rule.

In Berentsen, Menzio and Wright (2007), a very different model of the labor market – the one provided by Mortensen and Pissaridesis (1994) – is integrated into monetary economics. There are some advantages to that search-based model of unemployment, including the fact that it generates more interesting individual labor market histories, and also more interesting aggregate dynamics, because unemployment is a state variable (it takes time for individuals to find jobs). On the other hand, as is well known, the Mortensen-Pissaridesis model is analytically tractable only with linear utility. Hence, one cannot say it provides an alternative to quasi-linearity for monetary economics, and one certainly could not attempt to prove theorems like the ones in this paper, about how the slope of the Phillips curve depends on cross derivatives of the utility function. But it is good to know that both of these models of the labor market can easily be generalized to accommodate monetary exchange.

References

[1] Aiyagari, S. Rao and Neil Wallace (1991) "Existence of Steady States with Positive Consumption in the Kiyotaki-Wright Model," *Review of Economic Studies* 58, 901-916.

[2] Aliprantis C., G. Camera and D. Puzzello (2007) "Anonymous markets and monetary trading," forthcoming in *Journal of Monetary Economics*.

[3] Aruoba, S. Boragan and Randall Wright (2003) "Search, Money, and Capital: A Neoclassical Dichotomy," *J. Money, Credit and Banking* 35, 1085-1105.

[4] Aruoba, S. Boragan, Chris Waller and Randall Wright (2005) "Money and Capital," *mimeo,* University of Pennsylvania.

[5] Aruoba, S. Boragan, Guillaume Rocheteau and Chris Waller (2007) "Bargaining in Monetary Economies," forthcoming in *Journal of Monetary Economics*.

[6] Berentsen, Aleksander, Guido Menzio and Randall Wright (2007) "Inflation and Unemployment in the Long Run," *mimeo,* University of Pennsylvania.

[7] Berentsen, Aleksander, Guillaume Rocheteau (2003) "Money and the Gains from Trade," *International Economic Review* 44, 263-297.

[8] Camera, Gabriele and Dean Corbae (1999) "Money and price dispersion," *International Economic Review* 40, 985-1008.

[9] Chiu, Jonathan and Miguel Molico (2007) "Liquidity and Welfare Costs of Inflation" *mimeo*, Bank of Canada.

[10] Christiano, Lawrence and Martin Eichenbaum (1992) "Current Real Business Cycle Theory and Aggregate Labor Market Fluctuations," *American Economic Review* 82, 430-450.

[11] Cooley, Thomas and Gary D. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *American Economic Review* 79, 733-748.

[12] Cooper, Russel (1987) *Wage and employment patterns in labor contracts: Microfoundations and macroeconomic applications*, Harwood Academic Publishers.

[13] Corbae, Dean, Ted Temzilides and Randall Wright (2003) "Directed Matching and Monetary Exchange," *Econometrica* 71, 731-56.

[14] Dutu, Richard, Benoit Julien and Ian King (2007) "Competing Auctions in a Monetary Economy," *mimeo*, University of Aukland.

[15] Faig, Miguel "Divisible Money in an Economy with Villages," *mimeo*, University of Toronto.

[16] Galenianos, Manolis and Philipp Kircher (2006) "Multilateral Matching and Monetary Exchange," *mimeo*, University of Pennsylvania.

[17] Garratt, Rod, Todd Keister and Karl Shell (2004) "Comparing Sunspot Equilibrium and Lottery Equilibrium Allocations: The Finite Case," *International Economic Review* 45, 351-386.

[18] Green, Edward J. and Ruilin Zhou (1998) "A rudimentary random-matching model with divisible money and prices," *Journal of Economic Theory* 81, 252-271.

[19] Hansen, Gary D. (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-327.

[20] Kiyotaki, Nobuhiro and Randall Wright (1989) "On money as a medium of exchange," *Journal of Political Economy* 97, 927-954.

[21] Kiyotaki, Nobuhiro and Randall Wright (1993) "A Search-Theoretic Approach to Monetary Economics," *American Economic Review*, 83 63-77.

[22] Kocherlakota, Narayana (1998) "Money is Memory," *Journal of Economic Theory* 81, 232-251.

[23] Kydland, Finn E. (1994) "Business Cycles and Aggregate Labor Market Fluctuations," in *Frontiers of Business Cycle Research*, ed. by Thomas F. Cooley, Princeton University Press.

[24] Lagos, Ricardo and Randall Wright (2003) "Dynamics, Cycles and Sunspot Equilibria in Genuinely Dynamic, Fundamentally Disaggregative Models of Money," *Journal of Economic Theory* 109, 156-71.

[25] Lagos, Ricardo and Randall Wright (2005) "A unified framework for monetary theory and policy analysis," *Journal of Political Economy* 113, 463-84.

[26] Lucas, Robert and Prescott, Edward (1974). "Equilibrium search and unemployment," *Journal of Economic Theory* 7, 188-209.

[27] Moen, Espen R. (1997) "Competitive search equilibrium." *Journal of Political Economy* 105, 385-411.

[28] Molico, Miguel (2006) "The distribution of money and prices in search equilibrium," *International Economic Review* 46, 701-722.

[29] Mortensen, Dale and Christopher Pissarides (1994) "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*.

[30] Prescott, Edward C., Richard Rogerson and Johanna Wallenius (2006) "Aggregate Labor Supply: A Statement about Preferences, Technology, and their Interaction," *mimeo*, Arizona State University.

[31] Rocheteau, Guillaume and Randall Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

[32] Rocheteau, Guillaume, Peter Rupert, Karl Shell and Randall Wright (2006) "General Equilibrium with Nonconvexities and Money," *Journal of Economic Theory*, in press.

[33] Rocheteau, Guillaume, Peter Rupert, and Randall Wright (2007) "Inflation and Unemployment in General Equilibrium," *mimeo*.

[34] Rogerson, Richard (1988) "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics* 21, 3-16.

[35] Rogerson, Richard and Randall Wright (1988) "Involuntary unemployment in economies with efficient risk sharing," *Journal of Monetary Economics* 22, 501-515.

[36] Sargent, Thomas J. (1979) *Macroeconomic Theory*, New York: Academic Press.

[37] Sargent, Thomas J. and Lars Ljungqvist (2006) "Indivisible Labor, Human Capital, Lotteries, and Personal Savings: Do Taxes Explain European Employment?" *mimeo*.

[38] Shell, Karl and Randall Wright (1993) "Indivisibilities, lotteries, and sunspot equilibria," *Economic Theory* 3, 1-17.

[39] Shi, Shouyong (1995) "Money and prices: A model of search and bargaining," *Journal of Economic Theory* 67, 467-496.

[40] Shi, Shouyong (1997) "A divisible search model of fiat money," *Econometrica* 65, 75-102.

[41] Shimer, Robert (1996) "Contracts in frictional labor market," *mimeo*, Princeton University.

[42] Stockman, Alan (1981) "Anticipated Inflation and the Capital Stock in a Cash-in-advance Economy," *Journal of Monetary Economics* 8, 387-393.

[43] Trejos, Alberto and Randall Wright (1995) "Search, bargaining, money, and prices," *Journal of Political Economy* 103, 118-141.

[44] Wallace, Neil (2001) "Whither Monetary Economics?" *International Economic Review* 42, 847-869.

[45] Zhou, Ruilin (1999) "Individual and aggregate real balances in a random-matching model," *International Economic Review* 40, 1009-1038.

[46] Zhu, Tao (2003) "Existence of a Monetary Steady State in a Matching Model: Indivisible Money," *Journal of Economic Theory* 112, 307-324.

[47] Zhu, Tao (2005) "Existence of a Monetary Steady State in a Matching Model: Divisible Money," *Journal of Economic Theory*, 123, 130-160.