INFLATION BAND TARGETING
AND
OPTIMAL INFLATION CONTRACTS

by

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1. Introduction

With the development of the literature on the time-inconsistency problem in Kydland and Prescott (1977), Calvo (1978) and Barro and Gordon (1983), there has been an increasing recognition both by policymakers and the academic literature that solving the time-inconsistency problem is crucial to the successful conduct of monetary policy. The time-inconsistency problem arises because there are incentives for the monetary policymaker to try to exploit the short-run tradeoff between employment and inflation to pursue short-run employment objectives using expansionary monetary policy, even though the result are poor long-run outcomes – higher inflation, with no benefit on the output front.

Two approaches have been suggested in the literature to cope with the time-inconsistency problem: appointment of a conservative central banker (Rogoff, 1985) or adoption of optimal contracts for monetary policymakers (Walsh, 1995). Although both these approaches have attractive theoretical properties, they are difficult to implement in practice. It is likely to be quite difficult to find a central banker with the "right" preferences and it is hard to believe that politicians would naturally want to appoint central bankers with different preferences than theirs. Also, an opportunistic government would also be unlikely to appoint a conservative central banker, so that a regime based on having a conservative central banker is unlikely to be stable over time.

Optimal inflation contracts are also infeasible because central bankers are not paid very highly, and this is particularly true in the United States, where the chairman of the Board of Governors is paid far less than many economics professors. Thus it is highly unlikely that governments would be willing to write an inflation contract that would give a central banker sufficient incentives to produce optimal policy. An optimal inflation contract also requires that the monetary costs inflicted by the contract on the central baker would have to be translatable into utility units. Furthermore, public officials are almost never paid on the basis of their performance and we know of no central banker anywhere in the world that has performance-based pay. In addition, writing an explicit inflation contract in which the central bank is rewarded for undershooting the optimal inflation rate is politically untenable.[1]

An alternative approach that has been adopted by many central banks is inflation band targeting, in which the central bank has a target range and the central bank bears some cost if inflation goes outside the range. For example, the Governor of the Reserve Bank of New Zealand is subject to dismissal if inflation falls outside the target range, while the Governor of the Bank of England must write a public letter to the government explaining when the inflation rate falls outside the plus or minus one percent range around the 2% inflation target. In this regard inflation band targeting seems to be closely related to a dismissal rule, as discussed by Walsh (2002). However, unlike the dismissal rule, inflation band targeting does not require knowledge of how much the central banker values his office.

Inflation band targeting has several advantages relative to either appointment of a conservative central banker or optimal inflation contracts. First it eliminates the problem of finding the perfect central banker with the right preferences. Second, the framework is likely to be stable over time once the government has agreed to it. Third the target range provides added flexibility to the inflation targeting regime that is more palatable to politicians. Fourth, it is a simple framework that is easily implemented, in contrast to optimal inflation contracts. Indeed, inflation band targeting.

Although inflation band targeting has these attractive features, to our knowledge there is no research that provides a theoretical treatment of how inflation target bands can be designed to mitigate the time-inconsistency problem.[2] It is not clear that inflation band targeting has desirable properties. Indeed, some economists, including one of the authors of this paper, have been skeptical of inflation band targeting because it might produce too much focus on the edges of the range that can lead to the central bank to concentrate too much on keeping the inflation rate just within the range, rather than hitting the midpoint of the range (Bernanke, Laubach, Mishkin and Posen, 1999, and Mishkin, 2001).

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[1] Svensson (1997) also point out that if a low inflation level is associated with high unemployment, then an inflation contract may be politically difficult to implement since it rewards a central banker when inflation is low.

[2] Amano, Black and Kasumovich (1997) examines whether the literature on exchange rate target zones can be applied to inflation band targeting. Erceg (2002) interpret the target range as a confidence interval of inflation derived from the preferences of the policymakers. The optimal bandwidth should then be chosen to reflect the desired point on the trade-off locus between inflation and unemployment volatility. Neither of these papers, however, consider inflation band targeting as a solution to the time-inconsistency problem.

In this paper we analyze how inflation target ranges work in the context of a Barro-Gordon (1983) type model, but which has a more realistic setting in that the time-inconsistency problem stems not from the preferences of the central bank, as in Barro-Gordon, but instead from political pressures from the government. We demonstrate that inflation target bands or a range can achieve many of the benefits of these other strategies, providing a possible reason why this strategy has been used by so many central banks. Our theoretical model also enables us to outline how an inflation targeting range should be designed optimally and how it should change when there are changes in the nature of shocks to the economy.

To analyze the properties of inflation band targeting, we first examine a canonical benchmark model of optimal monetary policy and then illustrate the time-inconsistency problem which derives from discretionary monetary policy when the preferences of the government differs from the central bank's. We then demonstrate what the optimal inflation contract would look like, and then develop the theory of inflation band targeting, showing how it replicates many of the features of the optimal inflation contract.

2. A Canonical, Benchmark Model of Optimal Monetary Policy

The economy is described by the expectation-augmented Phillips curve,

$$u_t = u^n + b\left(\pi_t^e - \pi_t\right) + \varepsilon_t \qquad (1)$$

where, u^n is the natural rate, u_t is the realized unemployment rate in period *t*, ε_t is an i.i.d supply shock with mean 0 and fixed variance σ_ε^2, and b is a positive constant. The realized inflation and expected inflation in period *t* are denoted by π_t and π_t^e, respectively. The public forms its expectations rationally, i.e.,

$$\pi_t^e = E_{t-1}\left[\pi_t\right] \qquad (2)$$

where E_{t-1} refers to rational expectations formed with the complete information set up until and including period *t-1*.

To keep the model simple we assume that the central bank directly sets the inflation rate. However, the monetary authorities cannot perfectly control the inflation level i.e.,

$$\pi_t = \pi_t^{cb} + z_t$$

where z is a normally distributed control error with mean 0 and a fixed variance σ_z^2. Additionally, the central bank determines the inflation rate after observing the public's expectations and the supply shock. Consequently, since the supply shock is not included in the public's information set at *t-1*, the central bank can mitigate supply shock effects on real activity by creating surprise inflation.

Following a large literature we assume that the social welfare function at time *t* is specified over the inflation and unemployment rate in the following manner,

$$W_t = -\frac{1}{2}\left[\omega_\pi \left(\pi_t - \pi^*\right)^2 + \omega_u \left(u_t - u^n\right)^2\right] \qquad (3)$$

where ω_π is the weight on inflation volatility around its socially optimal level π^*, and ω_u is the weight on unemployment volatility around its natural level.[3]

Now suppose that the monetary authorities could credibly commit to a state-contingent rule of the inflation rate. The optimal rule can then be derived by maximizing the sum of current and future discounted welfare levels subject to the Phillips curve and the condition that expectations are rational i.e.,

$$\underset{\pi_t}{Max} \quad E_t\left\{\sum_{i=0}^{\infty} \beta^i W_{t+i}\right\} \qquad \text{subject to equations (1) and (2)}$$

Since there are no endogenous state variables, the maximization problem collapses to a one period problem. The optimal inflation rate at time *t* can then be expressed as:

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[3] See Reis (2004) for a formal derivation of the Phillips-curve and the welfare function (3) based on a general equilibrium framework.

$$\pi_t^{op} = \pi^* + \left(\frac{\omega_u b}{\omega_\pi + \omega_u b^2} \right) \varepsilon_t \tag{4}$$

Equation (4) indicates that it is optimal to set the inflation rate equal to the social optimal level of inflation and to respond to shocks that forces the current level of unemployment away from its long-run level. The shock management depends crucially on the preference parameters, ω_u and ω_π. The higher the relative weight is on unemployment stability the more volatile is inflation. Combining (1) and (4), we can derive the equilibrium unemployment rate as:

$$u_t = u^n + \left(1 - \frac{\omega_u b^2}{\omega_\pi + \omega_u b^2} \right) \varepsilon_t - b z_t \tag{5}$$

According to (4) and (5), average inflation and unemployment are equal to their target values. By substitution (4) and (5) into the welfare function we can derive the unconditional expected welfare level in equilibrium as:

$$E[W] = -\frac{1}{2} \left[\omega_\pi Var(\pi) + \omega_u Var(u) \right] \tag{6}$$

where,

$$Var(\pi) = \left(\frac{\omega_u b}{\omega_\pi + \omega_u b^2} \right)^2 \sigma_\varepsilon^2 + \sigma_z^2 \tag{7}$$

$$Var(u) = \left(1 - \frac{\omega_u b^2}{\omega_\pi + \omega_u b^2} \right)^2 \sigma_\varepsilon^2 + b^2 \sigma_z^2 \tag{8}$$

Since equations (4)-(8) represents optimal monetary policy, we will use this case as a benchmark in the analysis that follows. Any deviation from the optimal inflation rule described in equation (4) must hence lead to a decline in welfare.

3. Monetary Policy Under Discretion When Objectives of the Government and the Monetary Authorities Differ

One undesirable feature of the Barro-Gordon (1983) model, first raised by McCallum (1995) and elaborated on by Mishkin (2000), is that the time-inconsistency problem by itself does not imply that a central bank will pursue expansionary monetary policy which leads to inflation. Simply by recognizing the problem that forward-looking expectations in the wage- and price-setting process creates for a strategy of pursuing expansionary monetary policy, monetary policymakers can decide to "just not do it" and avoid the time-inconsistency problem altogether.[4] Indeed, central bankers are fully aware of the time-inconsistency problem, but the time-inconsistency problem remains nonetheless because politicians are able to put pressure on central banks to pursue overly expansionary monetary policy.

In this paper, we modify the Barro-Gordon (1983) model to allow for a more realistic setting in which the central bank is unable to pursue optimal monetary policy because the government may be motivated, for political reasons, to maximize a different welfare function than (3). In particular we assume that the relevant utility function for the government can be described as follows,

$$U_t^g = -\frac{1}{2}\left[\omega_\pi\left(\pi_t - \pi^*\right)^2 + \left(\omega_u + \delta\right)\left(u_t - u^*\right)^2\right] \tag{9}$$

If $\delta > 0$, the government gains some extra utility from lowering unemployment *volatility* and if $u^* < u^n$ the government also gets some extra utility from lowering the unemployment *level* below its natural level. We argue that the reason for the government's utility function to differ from the social welfare function is that lower unemployment (output) volatility and lower unemployment both increase the probability that it will be reelected. Hence the government has two objectives; (1) maximize the public's welfare level and (2) increase the probability of being reelected. These objectives are not always aligned. For example, the

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[4] In the typical Barro-Gordon framework the monetary authorities are assumed to have an overly optimistic unemployment target (i.e., a target below the natural rate), which produces a desire to exploit the short-run trade off between inflation and employment and hence an on average higher then optimal inflation rate in equilibrium. As Blinder (1997) points out, the monetary authorities could simply chose to target the natural rate and thereby avoid the time-inconsistency problem.

government may prefer a more stable and low unemployment rate in order to signal to potential voters that it is more competent than its political rivals.[5] As Persson and Tabellini (2000) points out, this behavior is rational if agents cannot contemporaneously observe the inflationary effects of expansionary monetary policy, which is likely since inflation typically lags economic activity.

The preferences of the monetary authorities on the other hand are assumed to mirror the social welfare function. This is because we believe that they are better modeled as technocrats who do not gain utility from being overly expansionary. However, if the central bank is not fully independent, the government is likely to have some influence over monetary policy. To incorporate this notion we assume that the monetary authorities are maximizing the following objective function at time *t*,

$$U_t^{cb} = \lambda U_t^g + (1-\lambda)W_t \tag{10}$$

where $0 \le \lambda \le 1$, and reflects the degree to which monetary authorities cares about the governments utility level.[6] The parameter λ could be thought of as the level of political influence the government has on the central bank.[7]

Clearly, it would be socially optimal if the monetary authorities could credibly commit to the inflation rule described by equation (4). However, as Kydland and Prescott (1977) argues, no such commitment exists when policy is discretionary in nature. Consequently, the monetary authorities, knowing that the public is aware that they are unable to resist the political influence from the government, do not believe that they can credibly manipulate inflation expectations. That is, they take inflation expectations as given when

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[5] See Persson and Tabellini (2000) for a survey of this literature. Fair (1978) presents empirical evidence that the incumbent government is more likely to be reelected in periods of high economic growth.

[6] Cukierman (1992) models central bank independence. He considers two possibilities: (i) the government and the central bank has the same objective function but the politicians have a lower discount factor β than the central bank (ii) the government has a greater desire than the central bank to create surprise inflation. He then measures the degree of central bank independence as the weighted average of the government's and the central bank's objective functions.

[7] In our model λ is exogenously determined. However, it is quite possible that λ would be positively related to $u^n - u^*$ and δ. That is, the greater the preference misalignment between the government and the central bank, the more influence the government tries to exert over the central bank.

maximizing U_t^{cb}. The optimization problem facing the monetary authorities under discretion can then be described as follows:

$$\underset{\pi_t}{Max} \quad E_t\left\{\sum_{i=0}^{\infty}\beta^i U_{t+i}^{cb}\right\} \qquad \text{s.t.} \qquad \text{equation (1)}$$

where π_{t+i}^e, for all *i* are taken as given. Since there are no endogenous state variables the maximization problem collapses into a one period specification. The first order condition gives us the following reaction function of the central bank:

$$\pi_t^{cb} = \pi^* + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*) + \left(\frac{\omega_u + \lambda\delta}{\omega_\pi}\right) b\left(u_t - u^n\right) \qquad (11)$$

Equation (11) suggests that, under discretion, the monetary authorities have three concerns when determining π_t. The first is to set inflation as close as possible to the social optimal level of inflation, π^*, i.e., the long-run inflation target. The second concern is to appease the pressure from the government by raising the inflation rate above its long-run target in order to push the unemployment rate closer to u^*. As equation (11) shows, if the unemployment target of the government coincides with that of the central bank, the pressure on the central bank to inflate the economy disappears. The third concern of the monetary authorities is to stabilize unemployment around its natural rate. By adjusting the inflation rate the monetary authorities can mitigate the effect of a supply shock on real activity. Notice that the response to a supply shock is increasing in δ. The higher δ is, the greater is the pressure from the government on the central bank to stabilize real activity relative to inflation.

Let us solve for the long-run equilibrium levels of the inflation and unemployment under discretion. The private sector forms its expectations rationally but cannot observe the supply shock, ε_t. Taking the expectations of (11) using equation (1) we get the following expressions for the equilibrium inflation and unemployment rate:

$$\pi_t = \pi^* + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*) + \left(\frac{(\omega_u + \lambda\delta)b}{\omega_\pi + (\omega_u + \lambda\delta)b^2}\right)\varepsilon_t + z_t \qquad (12a)$$

$$u_t = u^n + \left(1 - \frac{(\omega_u + \lambda\delta)b^2}{\omega_\pi + (\omega_u + \lambda\delta)b^2}\right)\varepsilon_t - bz_t \qquad (12b)$$

Equation (12a) and (12b) tell us several interesting facts regarding the determinants of the level and volatility of equilibrium inflation and unemployment. First, the expected long-run unemployment rate is simply equal to the natural rate. Secondly, expected long-run inflation is above the social optimal level and equal to:

$$\pi_{LR} = \pi^* + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*) \qquad (13)$$

This so-called inflation bias arises in our model because the central bank is not fully independent (i.e., $0 < \lambda < 1$). That is, the monetary authorities will on average be pressured by the government to inflate the economy. The central bank will raise inflation until the marginal cost of pushing inflation above its socially optimal rate equals the perceived marginal benefits from giving in to political pressure. However, since the public behaves rationally it will expect a higher level of inflation and hence there is no impact on real activity on average. Figure 1 illustrates this result assuming that π^* = 2%. Inflation expectations are measured on the x-axis and the central banks optimal inflation rate is measured on the y-axis. The dotted 45-degree line represents all the feasible equilibrium points. The solid line represents the central banks optimal inflation level given the public's inflation expectations under discretion (assuming that the supply shock is zero). The intersection of these two schedules represents the steady state level of inflation. The figure indicates an equilibrium inflation of approximately 2.8% and thus a bias of 0.8%.

From (12a) and (12b) we can derive the long-run inflation and unemployment volatility under discretion as:

$$Var_D(\pi) = \left(\frac{(\omega_u + \lambda\delta)b}{\omega_\pi + (\omega_u + \lambda\delta)b^2} \right)^2 \sigma_\varepsilon^2 + \sigma_z^2 \tag{14a}$$

$$Var_D(u) = \left(1 - \frac{(\omega_u + \lambda\delta)b^2}{\omega_\pi + (\omega_u + \lambda\delta)b^2} \right)^2 \sigma_\varepsilon^2 + b^2\sigma_z^2 \tag{14b}$$

When comparing equations (14a) and (14b) with equations (7) and (8), we see that if $\lambda\delta > 0$, then inflation (unemployment) volatility is excessively high (low) under discretion. The excess inflation volatility is negatively related to the degree of central bank independence and positively related to how much the government cares about stabilizing real activity relative to the central bank. Figure 2 shows the central bank's optimal inflation rate (y-axis) given the level of the supply shock (x-axis). The dashed line represents the optimal rule derived in section 2 and the solid line represents the supply shock management under discretion. Note that the slope of the reaction function is flatter under the optimal rule; hence there is excess inflation volatility under discretion.[8]

Another interesting result is that our simple model predicts that the level of inflation is positively correlated with the volatility of inflation through the stabilization bias parameter, δ. Svensson (1997) derives a similar prediction by introducing employment persistence. In his model, the persistence insures that surprise inflation has a positive effect on employment in the future and thus provides an additional incentive for the policymakers to both be overly expansionary *and* react stronger to supply shocks. In equilibrium this causes a larger inflation bias and more volatile inflation than in the absence of employment persistence. In contrast, the positive relationship between the level and volatility of inflation in our model stems entirely from the government's desire to push unemployment volatility below its socially optimal level.

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[8] Svensson (1997) also finds inflation volatility too high under discretion. However, the source of the distortion stems from persistence in employment. If supply shocks have a persistent effect on employment, then there is an extra incentive for the monetary authorities to mitigate the effect of the supply shock on real activity. Clarida, Gali and Gertler (1999) examine discretionary monetary policy in a New Keynesian framework and find that persistence in the cost-push shock leads to excess inflation volatility. This, however, is due to the forward looking behavior of price setters in the economy which is absent in our and Svensson's (1997) models.

Given equations (13) and (14), the unconditional expected welfare level in equilibrium can be expressed as:

$$E[W] = -\frac{1}{2}\left[\omega_\pi\left(\pi_{LR} - \pi^*\right)^2 + \omega_\pi Var_D(\pi) + \omega_u Var_D(u)\right] \qquad (15)$$

Comparing (15) to (6) we can identify three determinants of the welfare distortions under discretion:

(i) *The government's unemployment target,* u^*: Given the level of independence, the more misaligned the government's unemployment target is with the natural level, the higher is the long-run inflation bias and the lower is welfare.

(ii) *The government's stabilization bias,* δ: Given the level of independence, the government's excessive weight on unemployment stability has two negative effects on overall welfare. First, the more misaligned the government is with the monetary authorities regarding the stabilization objective the smaller is the unemployment volatility and hence the greater is inflation volatility. Second, the inflation bias is positively related to δ. This stems from the cost structure of the welfare function. The higher δ is, the higher is the perceived cost by the government of any given difference between the natural rate and the government's unemployment target and the more pressure it will lever on the monetary authorities to be overly expansionary

(iii) *Degree of political influence*: Since incomplete central bank independence allows the government to affect monetary policy decisions, the degree of political influence affects *both* the inflation bias and inflation volatility. That is, an increase in political influence causes a rise in the average level and volatility of inflation. This positive relationship is consistent with empirical evidence (see Alesina and Summers (1993) and Franzese (1998)).

3.1 Complete Independence under Discretion

The previous section suggests that a simple way of reducing the excess inflation level and volatility is to decrease the influence that the government has over monetary policy i.e., reducing λ. It is instructive to consider the case when the central bank is completely independent ($\lambda = 0$). In this case the reaction function (12) of the monetary authorities is reduced to:

$$\pi_t^{cb} = \pi^* + \left(\frac{\omega_u}{\omega_\pi}\right) b\left(u_t - u^n\right) \tag{16}$$

The expression shows that the monetary authorities no longer have a desire to on average push inflation above its social optimal level. Additionally, the stabilization bias is not present anymore. In a sense the technocrats are now left alone to conduce monetary policy. Taking the expectation of (16) and using equation (1), it is clear that equilibrium inflation and unemployment are now equal to equation (4) and (5), respectively. Hence if the central bank is completely independent, then it does not matter whether monetary policy is discretionary in nature or simply follow a pre-specified rule. The optimal inflation path will be implemented. This result arises because the public is aware that, even thought monetary policy is discretionary, there is no incentive for the monetary authorities to systematically exploit the short-run relation between unemployment and inflation. Hence, no commitment is necessary.

It will be convenient to view equation (16), as the optimal reaction function under discretion. That is, if it is impossible to eliminate the political influence on monetary policy, then other institutional designs which transforms (12) into (5) should be considered.

4. Optimal Inflation Contracts

In order to understand the relationship between inflation band targeting and optimal inflation contracts, we need to examine the approach to reduce the welfare distortions under discretion suggested by Walsh (1995) and Persson and Tabellini (1993). The idea is to write a contract between the government and the monetary authorities which penalizes the monetary authorities if they deviate from the optimal level of inflation. Most of the literature considers *linear* contracts that inflict a constant marginal penalty on the central bank when it raises inflation above its optimal level. As Svensson (1997) shows, these contracts are equivalent to the case of appointment of a target-conservative central banker.[9] We will now show that a *quadratic* inflation contract can eliminate both the inflation bias and the excess volatility in our model. Additionally, this quadratic inflation contract is equivalent to the combination of a target-conservative and weight-conservative that we describe in appendix A.

Define the quadratic inflation contract as an added penalty structure to the central banks objective function U_t^{cb} defined by equation (10) i.e.,

$$U_t^{cb} - \alpha\left(\pi - \pi^*\right) - \frac{\beta}{2}\left(\pi - \pi^*\right)^2 \tag{17}$$

The first linear term of the contract inflicts a positive constant marginal cost on the monetary authorities when inflation is raised above the socially optimal level and thus can undo the inflation bias. The second quadratic term of the contract inflicts an increasing marginal cost and will hence discourage the monetary authorities from reacting too strongly to supply shocks and thus can undo the stabilization bias. The first order condition of the maximization problem can be expressed as:

$$\frac{\partial U_t^{cb}}{\partial \pi_t^{cb}} - \left[\alpha + \beta\left(\pi - \pi^*\right)\right] = 0 \tag{18}$$

The second term of expression (18) is the marginal cost that the quadratic contract inflicts on the monetary authorities. Deriving the new reaction function from (18) we have:

$$\pi_t^{cb} = \pi^* + \left[\frac{\left(\omega_u + \delta\right)\lambda b(u_t^n - u^*) - \alpha}{\omega_\pi + \beta}\right] + \left(\frac{\omega_u + \lambda\delta}{\omega_\pi + \beta}\right) b\left(u_t - u^n\right) \tag{19}$$

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[9] We refer to a target-conservative central banker as one that has an inflation target lower than the optimal rate. Svensson (1997) does not use the term “target-conservative central banker.” Instead, he refers to this set-up as a constant inflation targeting regime.

Clearly the optimal parameters of the inflation contract must transform (19) into (16). This is true under the following parameterization:

$$\alpha = (\omega_u + \delta)\lambda b(u^n - u^*) \qquad \text{and,} \qquad \beta = \lambda\delta\left(\frac{\omega_\pi}{\omega_u}\right)$$

Figure 3 shows the relationship between the marginal cost imposed by the optimal inflation contract on the monetary authorities for different levels of inflation (see the dashed line). As indicated by expression (18) the marginal cost is positive at the optimal inflation level and increases linearly as inflation rises. In the appendix we show that this solution is indeed equivalent to the optimal conservative central banker.

5. Modeling Inflation Band Targeting

Inflation band targeting can be thought of as a simpler form of an inflation contract. In inflation band targeting the government and the monetary authorities agree upon an inflation *range* within which inflation must be kept, and if the central bank fails to contain inflation within the specified range some form of punishment is inflicted on the monetary authorities. There are two main components of inflation band targeting; (1) the specified range and (2) the degree of accountability. Let us denote the upper and lower boundaries of the target range by $\overline{\pi}$ and $\underline{\pi}$, respectively, and the cost of overshooting (undershooting) the range by C.[10] Because the monetary authorities cannot perfectly control inflation there will always be a positive probability of failing to hit the target range. The new objective function facing the central bank at time *t* can thus be written as:

$$U_t^{cb} - C\{1 - F(\overline{\pi} - \pi)\} - C\{F(\underline{\pi} - \pi)\} \qquad (20)$$

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[10] It is also possible for the perceived cost of overshooting the range to be higher than to undershoot it. However, we do not entertain that possibility in the analysis presented in this version of the paper.

where U_t^{cb} is defined by (10) , $F(\bullet)$ is the cumulative distribution function of the control error z_t , $\overline{\pi} = \pi^* + BW$, and $\underline{\pi} = \pi^* - BW$. For example, an inflation band target of 1-3% can be described by $\pi^* = 2$ and a bandwidth of $BW = 1$. The F.O.C can be derived as:

$$\frac{\partial U_t^{cb}}{\partial \pi_t^{cb}} - C\left[f(\pi^* + BW - \pi) - f(\pi^* - BW - \pi)\right] = 0 \qquad (21)$$

The first term of expression (21) corresponds to the F.O.C under discretion. The second term is the marginal cost of missing the target range. Comparing (18) and (21) we can see how similar the set up is to an inflation contract. The difference is simply the functional form of the marginal cost structure. Unfortunately, because we assume that the control error is normally distributed there is no closed form solution for the reaction function of the central bank. Hence, in the proceeding analysis we rely upon simulations in order to understand the effects that the inflation range has on the long-run inflation level and volatility. [11]

The parameter values chosen for the simulation exercises are based on U.S. annual inflation and unemployment data. The slope of the Philips curve, *b,* is equal to 0.64 as suggested by Reis (2004). The standard deviations of the control error and supply shock are set at 0.6 and 1, respectively.[12] The natural rate is assumed to be 4% and the optimal level of inflation is set at 2%.[13] The preference parameters of the social welfare function and the

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[11] The first order condition is derived under the assumption of i.i.d supply shocks. We do this to make our analysis of the inflation band target more tractable. One way in which previous literature has incorporated persistence in the unemployment rate is by including a lagged unemployment rate in the Phillips curve (e.g., Svensson (1997)). This would not change our result substantially as long as inflation stays within the target range. However, if the supply shock is large, the persistence in unemployment would cause inflation to be outside the range for several periods. In this case the central bank would pay the cost (C) every period until inflation converges within the target range. In reality, this repeated violation may cause the public to doubt the commitment by the central bank. Consequently, it may be optimal for the central bank to specify a state contingent inflation target range as long as inflation is outside the original target band. Indeed, this is exactly what occurred in 2003 in Brazil. Alternatively, if a country is adopting an inflation band targeting regime while inflation is well above its long-run target, such as in Canada and Chile, then it may be optimal to start with a appropriately specified midpoint target and then continuously adjust it until it converges to the desired long-run rate.

[12] Using survey data on inflation expectations since 1978 (provided by the University of Michigan's Research Survey Center) and the implicit GDP deflator we estimate the volatility of the control error to be approximately 0.6. Our estimate for the volatility of the supply shock is approximately consistent with Hodrick-Prescott filtered annual average unemployment data since 1970.

[13] The value of the natural rate is probably a low estimate for the U.S. However, the *level* of the natural rate is not important in our simulations. It is the *difference* between the government's unemployment target and the

degree of political influence are obviously chosen in a more ad hoc manner. We assume that unemployment and inflation stabilization are given equal importance (i.e., $\omega_\pi = \omega_u = 1$) and that the degree of independence is 0.5. [14] The latter implies that the central bank puts an equal weight on the utility of the government and the social welfare level. We further assume that the government has an unemployment target of 3.5% (i.e., $u^n - u^* = 0.5\%$) and cares 5 times as much about controlling unemployment versus inflation volatility (i.e., $\delta = 4$). Obviously these numbers are not based on any empirical estimates, but they do not seem too unrealistic. The annual inflation bias amounts to approximately 0.8.

5.1 The Marginal Cost Structure

The key to inflation band targeting is to understand how the marginal cost structure affects monetary policy under discretion. Figure 3 shows the marginal cost structure for inflation band targeting as a function of the inflation level, assuming a normally distributed control error. The solid line represents the marginal cost under inflation band targeting while the dashed line the marginal cost of the optimal inflation contract. The bandwidth is set at 1.2% and that accountability level at 3.[15] There are several characteristics of the marginal cost structure under inflation band targeting worth pointing out:

(i) *Symmetric around the midpoint*: The marginal cost is symmetric around the midpoint of the target range. Since the band is centered around the social optimal level of inflation, the cost of over and undershooting the target range is the same. Hence the marginal cost must be zero at the midpoint. If the inflation rate is above (below) the midpoint, the marginal cost is positive (negative) and encourages the monetary

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natural rate that is relevant. The optimal inflation rate of 2% is chosen because several existing inflation targeting regimes centers the target range around this level

[14] The equal weights could be viewed as a reflection of the dual mandate that the Federal Reserve is working under. Other central banks such as the European Central Bank has a hierarchical mandate where price stability is its foremost objective, perhaps indicating a lower weight on unemployment volatility relative to inflation volatility. The measure of political influence is consistent with Cukierman's (1992) estimate of the legal independence of the Fed during the 80s. Cukierman's index ranges from zero (minimal independence) to one (maximium independence). He estimates the overall legal independence of the Fed to be 0.51.

[15] These values were chosen to provide the clearest picture of the marginal cost structure under inflation targeting regime. As Table 1(b) shows, the optimal design corresponds to a bandwidth of 1.2% and a level of accountability of 5.

authorities to lower (raise) inflation. Consequently, inflation band targeting reduces the incentives of the monetary authorities to give in to political pressure and over-stabilize real activity. Additionally, since the marginal cost of increasing inflation above its socially optimal level is relatively higher than under discretion, the inflation bias must be lower. However, because the marginal cost is zero at the midpoint it is impossible to completely eliminate the inflation bias. Notice that this is in contrast to the inflation contract which allows for a positive marginal cost at the socially optimal level of inflation and can therefore completely eliminate the inflation bias.

(ii) *Increasing marginal cost inside the range:* The marginal cost of missing the target range increases as inflation moves away from the midpoint and towards the edges of the range. It peaks approximately at the boundaries. [16] Hence, similar to the optimal inflation contract, the slope of the marginal cost structure is positive within the target range. However, in contrast to the optimal inflation contract, the normality of the error term makes the marginal non-linear and therefore prevents the regime from completely eliminating the excess inflation level and volatility within the target range.[17]

(iii) *Decreasing marginal cost outside the range*: As inflation goes beyond the bandwidth the absolute value of the marginal cost decreases.[18] The diminishing marginal cost outside the range is due to the probability distribution of the control error. The further away the optimal inflation rate is from the edges of the target range the less likely it is for the realized inflation to fall within the target range. That is, the marginal cost of exiting the target range is close to zero when inflation is well outside the target range and will therefore have no impact on monetary policy. For example, if the economy is

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[16] The maximum marginal cost occurs when

$$\pi = \pi^* + BW\left[1 + e^{\frac{2}{\sigma^2}BW\left(\pi^* - \pi\right)} \Big/ 1 - e^{\frac{2}{\sigma^2}BW\left(\pi^* - \pi\right)}\right] .$$

Hence the maximum marginal cost occurs outside the boundaries of the target range. However, as BW / σ becomes large the maximum marginal cost approaches the boundary .

[17] In Appendix B we show that under a quadratic distribution of the control error the marginal cost structure is indeed linear within the target range.

[18] As indicated in the previous footnote, the marginal cost is actually *increasing* within a small interval immediately outside the boundaries. However, when the bandwidth is reasonable large relative to the control error this interval becomes very small.

hit by a large supply shock, the central bank might find it optimal to set the inflation rate outside the target range. At that point there is no incentive for the monetary authorities to restrain themselves from giving in to political pressure. As a result, the inflation level and volatility *outside* the target range remain close to their levels under discretion. As is clear from figure 3, the optimal inflation contract does not suffer from this drawback since the slope of the marginal cost structure does not depend on how far inflation is from its optimal level.

From the preceding discussion, it seems natural to design the optimal inflation band target in such a way that it approximates the marginal cost structure of the optimal inflation contract. The set-up presented in the previous section identifies two determinants of the shape of the marginal cost schedule.

(i) *Bandwidth (BW)*: Figure 4(a) shows the effect that a change in the bandwidth has on the marginal cost schedule. An increase in the bandwidth flattens the marginal cost around the midpoint, while a decrease has the opposite effect. An increase in the bandwidth would hence increase the incentives within the target range to give into political pressure.[19]

(ii) *Accountability* (C): Figure 4(b) shows how a change in the degree of accountability affects the marginal cost structure. An increase in the cost of missing the target range increases the marginal cost proportionally at any given inflation level. Hence, increasing the degree of accountability should make the monetary authorities less inclined to give into political pressure.

5.2 Reaction Function of the Central Bank under Inflation Band Targeting

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[19] Interestingly, if the bandwidth is smaller than one standard deviation of the control error i.e $BW < \sigma_Z$, then the slope of the marginal cost structure (close to the midpoint) is increasing when the bandwidth is widened. This is due to the increasing absolute value of the slope of the normal PDF that occurs within one std away from its mean.

To show how inflation band targeting affects equilibrium inflation and volatility we proceed by simulating the reaction function of the central bank. Figure 5 shows the central bank's optimal inflation rate as a function of expected inflation. The solid and dashed lines represent the reaction function under inflation band targeting and pure discretion, respectively. Again, the bandwidth is 1.2% and the degree of accountability is 3. The supply shock is assumed to be zero. The first thing to notice is that the central bank has a relatively more muted response to changes in expected inflation *within* the target range than under discretion. This is because the target range imposes an additional marginal cost of raising inflation above the midpoint. The public understands this and expects a lower inflation level than under discretion. The lower equilibrium level of inflation is represented by the intersection between the solid line and the dotted 45-degree line.

Figure 6 shows the central bank's desired inflation rate as a function of the realized supply shock. The solid and dashed line represents the reaction function under inflation band targeting and pure discretion, respectively. In order to focus on the effect that the target range has on shock management we eliminate the inflation bias by setting the government's unemployment target equal to the natural rate. The response of the monetary authorities to a given supply shock is clearly lower *within* the target range relative to pure discretion. Indeed, the supply shock response within the range is approximately the same as under the optimal rule. However, if the supply shock is large such that the desired inflation rate is *outside* the range, then the response by the monetary authorities is similar to that under pure discretion. This means that inflation band targeting reduces inflation volatility within but less so *outside* the target range.

5.3 Optimal Bandwidth and Accountability

Figures 7(a)-(d) show the effect that the bandwidth has on welfare, equilibrium inflation and inflation/unemployment volatility, respectively. Figure 7(d) illustrates that a narrow bandwidth may result in high inflation volatility since the probability of missing the inflation target is great. On the other hand, a wide bandwidth does little to reduce volatility *within* the target range and hence can also generate high inflation volatility. Figure 7(b) and 7(d) therefore suggested that, given an initial narrow (wide) bandwidth, a widening of the bandwidth may indeed reduce (increase) inflation volatility and increase (decrease) unemployment volatility. Figure 7(c) depicts the relationship between the bandwidth and equilibrium inflation. A narrow bandwidth suggests a higher marginal cost, given the inflation level, and thus should reduce the inflation bias. However, if the bandwidth is too narrow the probability of missing the target range may be so high that the inflation bias is greater than under a wider bandwidth. This is because inflation *outside* the target range remains close to its level under discretion. Hence an increase in the probability of missing the target range causes a rise in inflation expectations and forces the monetary authorities to increase inflation on average in order to alleviate the downward pressure on real activity.[20] Figure 7(c) also highlights that the inflation bias can never be fully eliminated with a symmetric inflation band target. As mentioned earlier, this is because the marginal cost due to the target range is zero at the social optimal level of inflation.[21] Figure 7(a) shows the resulting welfare level under various bandwidths. A too narrow bandwidth results in a high probability of missing the target range and will hence produce a high level and volatility of inflation. On the other hand, a too wide bandwidth, while decreasing the probability of missing the target range, does little to reduce the inflation level and volatility *within* the target range. Additionally, at the optimal level of the bandwidth (i.e., 1.2), the welfare level is not very sensitive to changes in the bandwidth. This suggests that the cost of deviating from the optimal bandwidth may not be very large. Indeed, this may be one reason why inflation band targeting is an attractive regime to implement in practice.

Figures 8(a)-(d) show the effect that the degree of accountability has on welfare, equilibrium inflation and inflation/unemployment volatility, respectively. Not surprisingly, figures 8(c) and 8(d) show that an increase accountability reduces both the inflation level and inflation volatility. Of course, a higher level of accountability also increases unemployment volatility. Consequently, as shown in figure 8(a), there exists a welfare maximizing level of accountability that trade off the reduction in the inflation bias and volatility against a higher level of unemployment volatility.

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[20] Additionally when the bandwidth is below one standard deviation of the control error, any further reduction in the bandwidth decreases the slope of the marginal cost curve and hence causes inflation to rise. Also notice that the equilibrium inflation rate is outside the target range for bandwidths that are extremely narrow (see section 5.6.

[21] If the midpoint is allowed to be lower than the social optimal level or the level of accountability at the upper boundary is higher than that of the lower boundary, then it is possible to eliminate the bias completely.

Table 1(a) shows the inflation level, inflation/unemployment volatility, and welfare under discretion and under the optimal rule/optimal contract for various values of δ and u^*. The inflation level under discretion varies from 2% to 4.24% while the standard deviations of inflation and unemployment lie within the intervals of 0.92-1.14% and 0.57-0.67%, respectively. The optimal rule (and the equivalently optimal inflation contract) implies an inflation and unemployment volatility of approximately 0.75 and 0.81 standard deviations, respectively. Table 1(b) shows the inflation level, inflation/unemployment volatility, and welfare under the optimal inflation band targeting regime. It also shows the corresponding optimal values of the bandwidth and degree of accountability.

From these simulations we can see that the inflation band targeting regime does very well in reducing both the inflation bias and the excess volatility. Of course, as mentioned in section 5.1, it cannot completely eliminate the welfare losses that arise under full discretion. For example, the lowest welfare level under discretion is -3.31 ($\delta = 6$ and $u^* = 3$). This is a welfare loss of 2.7 relative to the optimal rule/inflation contract. The optimal inflation band target increases the welfare level to -0.67 which is only 0.06 lower than the optimal rule/inflation contract. This corresponds to approximately a 98% reduction of the initial welfare loss.

Table 1(b) shows that the inflation volatility under the optimal inflation targeting regime is lower than under the optimal rule/inflation contract while the opposite is true for unemployment volatility. The reason is that the band target attempts to eliminate both the excess volatility and the inflation bias. However, at the point where the excess inflation volatility is eliminated, the marginal benefits from reducing the inflation bias is still greater than the marginal cost of over-stabilize inflation. Hence inflation (unemployment) volatility will be below (above) its optimal level.

Table 1(b) also tells us how to design an optimal inflation band target under various values of δ and u^*.The bandwidth only varies between 0.9 to 1.4 while the degree of accountability varies between 2.1 to 8.8. Furthermore, the optimal bandwidth does not seem to be affected much by the degree of the governments volatility bias, i.e., δ. Indeed, this distortion is most effectively reduced by a high degree of accountability since it reduces both the inflation bias and volatility. However, the simulations seem to suggest that a country where the government has a relatively low unemployment target (i.e., a low u^*) should have both a tighter band and a higher degree of accountability.

Finally, in order to asses the likelihood of missing the target range, it may be convenient to express the optimal bandwidth in terms of standard deviation of inflation. The final matrix in table 1(b) shows that there is little variation in this measure across our set of simulation. Indeed, it varies between 1.39 and 1.85. Assuming that the inflation rate is at the midpoint and that the inflation distribution is approximately normal, these numbers would indicate that the optimal inflation band targeting regime implies a 5-17% probability that the target range will be breached.[22] Intuitively this makes sense since the marginal cost structure is fairly close to that of the optimal inflation contract as long as realized inflation is not too far outside the target range.

Indeed, these numbers for the optimal bandwidth are not out of line with what we see in countries that have adopted inflation band targets. In a sample of fourteen countries in which there has been a stable operating framework for at least two years and where the country was not in the middle of an ongoing adjustment from high to low inflation, the average ratio of bandwidth to standard deviation is 1.7, well within the range of what our simulations find.[23]

5.4 Supply Shocks and Optimal Design

Because of the quadratic form of the welfare function any theoretical solution which completely eliminates the welfare distortions has to be characterized by a linear marginal cost structure. This means that the level of supply shock volatility becomes irrelevant to the optimal design of the inflation contract. One interesting feature of the inflation band targeting set-up is that its optimal design is sensitive to the level of supply shock volatility.

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[22] The distribution of inflation will not be normally distributed under inflation band targeting, but will have thicker tails because of the non-linear cost structure. However, the normal distribution is still likely to provide a good approximation.

[23] The data was collected from the IFS and individual central banks. The fourteen countries included in the sample are Australia, Canada, Chile, Czech Republic, Israel, Iceland Korea, New Zealand, Norway, Peru, Sweden Switzerland, Thailand, and U.K. Note however, the variation in this measure for individual countries is quite high, ranging from 0.4 to 3.9.   One problem with this calculation (because it is for individual countries over very short sample periods) is that the standard deviation of inflation for the particular sample period may not accurately reflect the unconditional standard deviation because the sample period is unusual.

Table 2, shows how the optimal design of inflation band targeting changes as the volatility of the supply shock increases. The basic intuition for changing the width of the inflation bands and the degree of accountability is that higher supply shock volatility makes it more likely that inflation will end up outside the target range. As the first two rows of table 2 show, in this case it becomes optimal to increase both the bandwidth and the degree of accountability. The widening of the bandwidth makes it more likely that inflation will be contained within the boundaries of the target range. On the other hand, the inflation volatility within the target range increases. The increase in the degree accountability reduces this problem by making it increasingly costly to deviate from the socially optimal level of inflation i.e., the slope of the marginal cost structure within the target range increases. It is also worth pointing out that the ability of the inflation band targeting regime to reduce welfare distortions is lower at higher levels of supply shock volatility.

One potentially important implication of table 2 is that developing countries, which typically are characterized by greater supply shock volatility, should implement an inflation band targeting regime with a high degree of accountability and a wide target range. In addition, they will benefit less from an inflation band targeting regime relative to countries that are less prone to large supply shocks.

5.6 Central Bank Independence as a Mean of Reducing Welfare Distortions

As is clear from previous discussions, a reduction in the political influence over monetary policy (i.e., λ) would reduce the welfare distortions under discretion. There are potentially many ways to achieve such a reduction. First, the obvious way to limit political influence is to increase central bank autonomy. Second, an increased degree of transparency and better communication with the public by the central bank could potentially prevent the government from using monetary policy for reelection purposes. Third, legislation such as the Maastricht treaty could clarify the goals of monetary policy and make it harder for the government to influence monetary policy outcomes. Indeed, all these approaches to decrease λ seem to be present in the inflation band targeting regimes that has been implemented in reality. Since different countries are likely to allow different degree of political influence, it would be interesting to analyze how the optimal design would change as λ changes.

Table 3, displays the optimal degree of accountability and bandwidth, as well as welfare levels, for various levels of political influence. From our analysis of discretionary monetary policy, we know that the more influence the government has over monetary policy the higher is the equilibrium inflation level and volatility. Thus, as table 3 indicates, just by reducing λ, there is a substantial improvement in welfare even without the use of inflation band targeting. Therefore insulating the central bank from political influence through communication strategies, price stability mandates and legislated central bank independence are another important dimension of institutional design to improve monetary policy performance.

Since a higher degree of accountability reduces both the inflation level and volatility, it seems reasonable to expect that a country with a relatively less independent central bank should implement an inflation band framework with a relatively higher degree of accountability. Table 3 clearly shows that this implication holds. The relationship between political influence and the optimal bandwidth, however, is less obvious. Indeed, as we see in table 3, the optimal bandwidth does not seem to change much with changes in λ and does have a monotonic relationship with λ.

5.6 When does Inflation Band Targeting not help?

Inflation band targeting does not always improve the discretionary outcome. It may be the case that the equilibrium inflation rate lies outside the target range. In fact, figure 9 shows the case where the reaction function of the central bank intersects the 45-degree line at the same point as under discretion. This situation is likely to arise if the initial inflation bias is large, the target range is very narrow, and the degree of accountability very low. Under these circumstances the marginal cost of giving in to political pressure is not high enough to make the framework credible. Rational agents realizing this will still expect an inflation rate identical to that under discretion.

The optimal bandwidth of the target range is easy to calculate, so unless the designers of the inflation band target are incompetent, it is unlikely that they would choose a target range that is too narrow. However, there well may be limits on how high accountability can be set. We have seen an example of this in Europe with the implementation of the Growth and Stability Pact in which countries are supposed to be fined if they exceed the deficit limit of 3% of GDP. However, when countries have exceeded this deficit limit there has been a reluctance to fine them. It is true that there still has been a cost for these countries when they exceed the deficit limit, but it does illustrate that accountability may be limited because if a punishment is felt to be too draconian, it will not be imposed. This has an important implication. In order to avoid needing a level of accountability (*C*) that is unenforceable, the inflation bias needs to be kept low and this requires that the amount of political influence over the central bank (λ)needs to be limited. This analysis thus provides another reason why the measures to promote central bank autonomy mentioned in the previous section are important elements of a well-designed inflation targeting regime.

It is also possible that the there exists multiple equilibria. Figure 9(b) shows the same cases as in 9(a) but with a higher degree of accountability. The reaction function crosses the 45-degree line three times. The first equilibrium occurs within the target range but very close to the upper boundary. The second equilibrium occurs outside the target range but at a lower inflation level than the original inflation level under discretion. However, this equilibrium is not stable. If inflation expectations falls below the second equilibrium inflation will converge towards the equilibrium inside the target range. On the other hand, if for some reason expected inflation falls above the second equilibrium then inflation will converge to the initial inflation level under discretion.

Figure 9(c) shows how the framework in 9(a) and (b) can be made stable by increasing the degree of accountability further. The equilibrium inflation is well inside the target range and the there is no other equilibrium outside the target range.

6. Conclusion

In this paper we have examined how target ranges work in the context of a Barro-Gordon (1983) type model, but which has a more realistic setting in that the time-inconsistency problem stems not from the preferences of the central bank, as in Barro-Gordon, but instead from political pressures from the government. In contrast to conjectures in the literature that are skeptical about the benefits of inflation target ranges, we find that target ranges turn out to be an excellent way to cope with the time-inconsistency problem and provide incentives that get monetary policy to be very close to optimal policy in which the time-inconsistency problem is avoided altogether.

Our theoretical model also shows how an inflation targeting range should be set and how it should respond to changes in the nature of shocks to the economy. We find that inflation band targeting has a marginal cost structure that is very close to that of the optimal inflation contract as long as realized inflation is not too far outside the target range. This tells us that the target range has to be wide enough so that realized inflation ends up inside it most of the time, and this also tells us that the more uncertainty there is about the inflation process, the wider the target range has to be. Indeed, this is what we actually find in practice, where emerging market countries, which are more likely to have more uncertainty about inflation outcomes, tend to choose wider target ranges.

The theoretical framework here thus shows how inflation band targeting has desirable characteristics and why central banks who target inflation have adopted target ranges as part of their monetary policy framework.

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Appendix A: Analysis of Appointment of a Conservative Central Banker

One of the most prominent remedies that the literature has put forth in order to reduce the welfare distortions that arise under discretion is to appointment of a conservative banker. There are two ways in which a central banker can be conservative: (1) he can be *weight-conservative*, that is put a higher weight on inflation volatility than is socially optimal, (2) he can be *target-conservative*, that is have an inflation target below the social optimum.

Weight-Conservative Central Banker

The approach of the weight-conservative central banker was first suggested by Rogoff (1985). In his model, as in most of the literature on the inflation bias problem, he assumes that the monetary authorities are targeting a “too low” unemployment rate. In equilibrium, under rational expectations, the low unemployment target then leads to an inflation bias. Rogoff shows that by appointing a weight-conservative central banker the inflation bias can be reduced, but only at the expense of higher unemployment volatility. Since there is no initial excess inflation volatility in Rogoff’s model, the increased unemployment volatility is welfare reducing.[24]

A common criticism of this general framework is that central bankers are technocrats and do not have an inherent desire to push unemployment below its natural rate (see Blinder (1997)). Our model, however, is not subject to this criticism since we assume that the central bank is targeting the *right* level of unemployment. It is the government that has an overly optimistic unemployment target. Hence, it is only through the assumption of incomplete independence that the inflation bias arises. This distinction turns out to be important when analyzing the effects of the appointing a weight-conservative central banker.

Let us start by specifying the preferences of the weight conservative central banker as follows:

$$U_t^C = -\frac{1}{2}\left[\omega_\pi\left(\pi_t - \pi^*\right)^2 + \omega_u^C\left(u_t - u^n\right)^2\right] \qquad \text{(A1)}$$

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[24] Of course, when there is an initial excess inflation volatility (i.e., too low unemployment volatility), the increase in unemployment volatility is likely to be welfare improving (see Lockwood, Miller and Zang (1995)).

By Substituting U_t^C for W_t in equation (10) we get the new objective function of the central bank:

$$U_t^{cb} = \lambda U_t^g + (1-\lambda)U_t^C .$$

Solving the modified maximization problem facing the monetary authorities, we can derive the new reaction function as:

$$\pi_t^{cb} = \pi^* + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*) + \left(\frac{\omega_u + \lambda\delta + (1-\lambda)(\omega_u^C - \omega_u)}{\omega_\pi}\right)b(u_t - u^n) \qquad \text{(A2)}$$

From equation (A2) it is clear that ω_u^C only enters into the third term and will not impact the source of the inflation bias. Hence, a weight-conservative central banker reduces the inflation volatility, but it does *not* affect the inflation bias.

This result is different from Rogoff (1985) who finds that a weight-conservative banker is able to reduce the inflation bias. The reason for our result being different is that the government, not the central bank, advocates the low unemployment target. Hence, the political pressure to systematically exploit the short-run Phillips curve in order to push unemployment below its natural rate remains and so appointing a weight-conservative central banker does not reduce the inflation bias. We believe that our result is intuitively more sensible than Rogoff's. Ironically, since there is an initial excess inflation volatility in our model, the reduction in inflation volatility is welfare improving and not welfare reducing as it is in Rogoff's set-up.

Comparing equation (A2) to the optimal reaction function under discretion (16), we can derive the preferences of the optimal central banker (i.e., the weight, ω_u^C, which neutralizes the impact of the parameters λ and δ on the stabilization objective) as:

$$\omega_u^C = \omega_u - \frac{\lambda}{(1-\lambda)}\delta \qquad \text{(A3)}$$

From equation (A3) we can also see that there is a limit to how low the weight can be. That is, depending on δ and the degree of political oversight, λ, it might not be feasible to eliminate all the excess inflation volatility.

Target-Conservative Central Banker

Svensson (1997) shows that by appointing a central banker with an inflation target lower than the socially optimal level, the inflation bias can easily be eliminated. However, he also emphasizes that the target-conservative central banker does not affect the excess inflation volatility arising under discretion. . Unlike Svensson's model, where the source of the inflation bias is internal to the central bank (i.e., it is the central bank that has a "too low" unemployment target), in our set-up the inflation bias stems entirely from political influence over monetary policy.

To see the implication of this difference we denote the new inflation target by π^T, and express the central bank's new objective function as:

$$U_t^{cb} = \lambda U_t^g + (1-\lambda) U^{IT}{}_t \tag{A4}$$

where,

$$U_t^{IT} = -\frac{1}{2}\left[\omega_\pi \left(\pi_t - \pi^T\right)^2 + \omega_u \left(u_t - u^n\right)^2\right] \tag{A5}$$

and with U_t^g remaining equal to (9). Solving the modified maximization problem facing the monetary authorities, we can derive the new reaction function as:

$$\pi_t^{cb} = \pi^* + \left[(1-\lambda)\left(\pi^T - \pi^*\right) + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*)\right] + \left(\frac{\omega_u + \lambda\delta}{\omega_\pi}\right) b\left(u_t - u^n\right) \tag{A6}$$

From equation (A6), it is clear that π^T does not affect the stabilization objective of the monetary authorities. Hence, the target-conservative central banker can only reduced the inflation bias *not* the excess inflation volatility. Comparing (A6) to the optimal reaction function under discretion (16), it is then straightforward to derive the optimal inflation target as:

$$\pi^T = \pi^* - \left(\frac{1}{1-\lambda}\right)\frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*) \qquad \text{(A7)}$$

That is, the central bank's optimal inflation target is lower than the socially optimal inflation rate, i.e., $\pi^T < \pi^*$. Similarly, Svensson (1997) derives the optimal inflation target to equal the social optimal level of inflation less the initial inflation bias. A potentially important difference between the optimal inflation target derived by Svensson (1997) and equation (A7) is that the differential between the target and the social optimal level is, in our case, greater than the initial inflation bias. This stems from the separation of the preferences of the monetary authorities and the government. In our set up the monetary authorities have to be even more conservative in order to compensate for the lack of commitment by the government to the new inflation target.

Combining the Weight-Conservative and Target-Conservative Central Banker

When a weight-conservative central banker also is target-conservative, then it is possible to eliminate both welfare distortions existing under discretion. Under this scenario the new reaction function of the central bank equals:

$$\pi_t^{cb} = \pi^* + \left[(1-\lambda)\left(\pi^T - \pi^*\right) + \frac{(\omega_u + \delta)}{\omega_\pi}\lambda b(u^n - u^*)\right] + \left(\frac{\omega_u + \lambda\delta + (1-\lambda)\left(\omega_u^C - \omega_u\right)}{\omega_\pi}\right) b\left(u_t - u^n\right)$$

Inserting equation (A1) and (A7) into the expression above we get the optimal reaction function under discretion described in equation (16). Hence, the low inflation target and the low weight on unemployment volatility ensure that equilibrium inflation level and volatility are equal to their respective social optimal level.

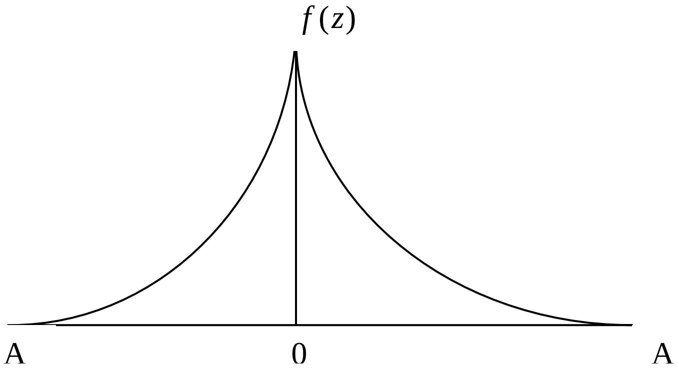
To see that the optimal weight-conservative and target-conservative central banker is equivalent to the optimal inflation contract discussed in the text, we can simply rewrite α and β as

$$\alpha = \omega_\pi \left(\pi^T - \pi^*\right) \qquad \text{and} \qquad \beta = \left(1-\lambda\right)\left(\frac{\omega_\pi}{\omega_u}\right)\left(\omega_u^C - \omega_u\right),$$

where π^T and ω_u^C are defined by equation (A3) and (A7).

Appendix B: Quadratic Distribution of the Control Error

If we assume that the control error has the following distribution



$$f(z) = \begin{cases} z > 0 & \frac{3}{2A} - \frac{3}{A^2} z + \frac{3}{2A^3} z^2 \\ z < 0 & \frac{3}{2A} + \frac{3}{A^2} z + \frac{3}{2A^3} z^2 \end{cases} \qquad \text{where} \quad -A \le z \le A \qquad \text{(B.1)}$$

Further assume that $A > BW$. That is, only when inflation is at or above (below) the upper (lower) boundary will the probability of undershooting (overshooting) be zero. We can now concentrate on the penalty structure within the target range.

$$MC_t = C\left\{ f\left(\overline{\pi} - \pi\right) - f\left(\underline{\pi} - \pi\right)\right\}$$

Using (B.1), and the fact that $\overline{\pi} = \pi^* + BW$ and $\underline{\pi} = \pi^* - BW$ we have:

$$\begin{aligned} MC_t &= C\left\{\left(\tfrac{3}{2A} - \tfrac{3}{A^2}\left(\overline{\pi} - \pi\right) + \tfrac{3}{2A^3}\left(\overline{\pi} - \pi\right)^2\right) - \left(\tfrac{3}{2A} + \tfrac{3}{A^2}\left(\underline{\pi} - \pi\right) + \tfrac{3}{2A^3}\left(\underline{\pi} - \pi\right)^2\right)\right\} \\ &= C\left\{- \tfrac{3}{A^2}\left[\left(\overline{\pi} - \pi\right) + \left(\underline{\pi} - \pi\right)\right] + \tfrac{3}{2A^3}\left[\left(\overline{\pi} - \pi\right)^2 - \left(\underline{\pi} - \pi\right)^2\right]\right\} \\ &= C\left\{- \tfrac{3}{A^2}\left(\pi^* - \pi\right) + \tfrac{3}{2A^3}\left[2\left(\pi^* - \pi\right)\left(\overline{\pi} - \underline{\pi}\right)\right]\right\} \\ &= C\left\{- \tfrac{6}{A^2}\left(\pi^* - \pi\right) + \tfrac{6}{A^3} BW\left[\left(\pi^* - \pi\right)\right]\right\} \end{aligned}$$

Hence we have that the marginal cost structure within the target range can be described as:

$$MC_t\left(\pi\right) = -\frac{6C}{A^2}\left\{1 - \frac{BW}{A}\right\}\left(\pi - \pi^*\right) \qquad \text{(B.2)}$$

Expression (B.2) bears out similar prediction in regards to the marginal cost structure as section 5.5. Firstly, it is symmetric around the optimal level of inflation. Secondly, since $A > BW$, an increase in the bandwidth reduces the absolute value of the marginal cost given inflation. Thirdly, an increase in accountability increases the absolute value of the marginal cost given inflation.

Table 1(a): Various Simulations under Discretion and Optimal Rule

Discretion

Inflation

δ

		2	3	4	5	6
u*	3	2.96	3.28	3.60	3.92	4.24
	3.25	2.72	2.96	3.20	2.96	3.68
	3.5	2.48	2.64	2.80	2.96	3.12
	3.75	2.24	2.32	2.40	2.48	2.56
	4	2.00	2.00	2.00	2.00	2.00

Inflation Volatility (std)

δ

		2	3	4	5	6
u*	3	0.92	0.99	1.05	1.10	1.14
	3.25	0.92	0.99	1.05	1.10	1.14
	3.5	0.92	0.99	1.05	1.10	1.14
	3.75	0.92	0.99	1.05	1.10	1.14
	4	0.92	0.99	1.05	1.10	1.14

Unemployment Volatility (std)

δ

		2	3	4	5	6
u*	3	0.67	0.63	0.59	0.56	0.54
	3.25	0.67	0.63	0.59	0.56	0.54
	3.5	0.67	0.63	0.59	0.56	0.54
	3.75	0.67	0.63	0.59	0.56	0.54
	4	0.67	0.63	0.59	0.56	0.54

Welfare

δ

		2	3	4	5	6
u*	3	-1.11	-1.51	-2.01	-2.60	-3.31
	3.25	-0.91	-1.15	-1.45	-1.22	-2.21
	3.5	-0.77	-0.89	-1.05	-1.22	-1.42
	3.75	-0.68	-0.80	-0.81	-0.88	-0.95
	4	-0.65	-0.69	-0.73	-0.76	-0.80

Optimal Rule/Contract

Inflation

δ

		2	3	4	5	6
u*	3	2.00	2.00	2.00	2.00	2.00
	3.25	2.00	2.00	2.00	2.00	2.00
	3.5	2.00	2.00	2.00	2.00	2.00
	3.75	2.00	2.00	2.00	2.00	2.00
	4	2.00	2.00	2.00	2.00	2.00

Inflation Volatility (std)

δ

		2	3	4	5	6
u*	3	0.75	0.75	0.75	0.75	0.75
	3.25	0.75	0.75	0.75	0.75	0.75
	3.5	0.75	0.75	0.75	0.75	0.75
	3.75	0.75	0.75	0.75	0.75	0.75
	4	0.75	0.75	0.75	0.75	0.75

Unemployment Volatility (std)

δ

		2	3	4	5	6
u*	3	0.81	0.81	0.81	0.81	0.81
	3.25	0.81	0.81	0.81	0.81	0.81
	3.5	0.81	0.81	0.81	0.81	0.81
	3.75	0.81	0.81	0.81	0.81	0.81
	4	0.81	0.81	0.81	0.81	0.81

Welfare

δ

		2	3	4	5	6
u*	3	-0.61	-0.61	-0.61	-0.61	-0.61
	3.25	-0.61	-0.61	-0.61	-0.61	-0.61
	3.5	-0.61	-0.61	-0.61	-0.61	-0.61
	3.75	-0.61	-0.61	-0.61	-0.61	-0.61
	4	-0.61	-0.61	-0.61	-0.61	-0.61

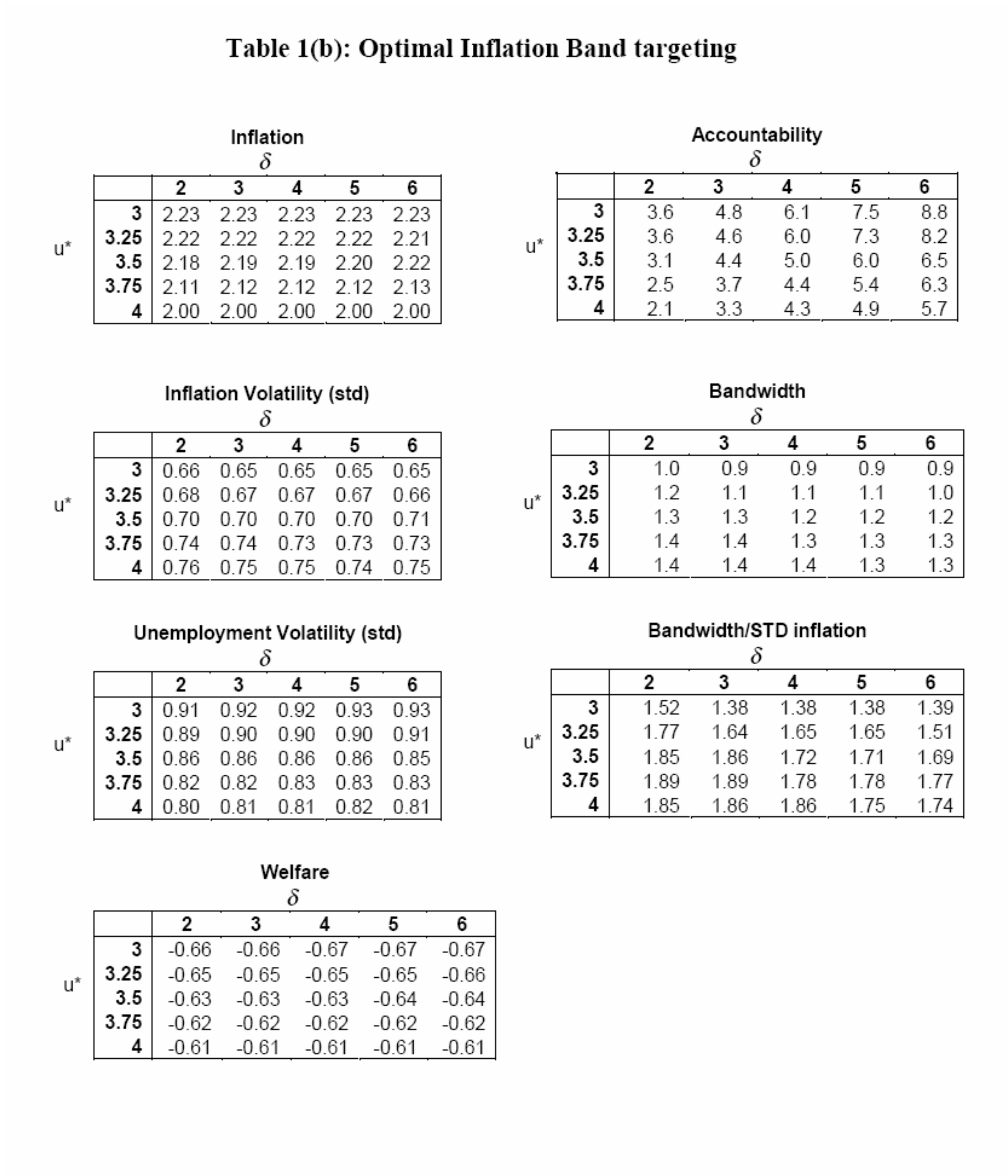


Table 1(b): Optimal Inflation Band targeting

Inflation

δ

		2	3	4	5	6
u*	**3**	2.23	2.23	2.23	2.23	2.23
	3.25	2.22	2.22	2.22	2.22	2.21
	3.5	2.18	2.19	2.19	2.20	2.22
	3.75	2.11	2.12	2.12	2.12	2.13
	4	2.00	2.00	2.00	2.00	2.00

Accountability

δ

		2	3	4	5	6
u*	**3**	3.6	4.8	6.1	7.5	8.8
	3.25	3.6	4.6	6.0	7.3	8.2
	3.5	3.1	4.4	5.0	6.0	6.5
	3.75	2.5	3.7	4.4	5.4	6.3
	4	2.1	3.3	4.3	4.9	5.7

Inflation Volatility (std)

δ

		2	3	4	5	6
u*	**3**	0.66	0.65	0.65	0.65	0.65
	3.25	0.68	0.67	0.67	0.67	0.66
	3.5	0.70	0.70	0.70	0.70	0.71
	3.75	0.74	0.74	0.73	0.73	0.73
	4	0.76	0.75	0.75	0.74	0.75

Bandwidth

δ

		2	3	4	5	6
u*	**3**	1.0	0.9	0.9	0.9	0.9
	3.25	1.2	1.1	1.1	1.1	1.0
	3.5	1.3	1.3	1.2	1.2	1.2
	3.75	1.4	1.4	1.3	1.3	1.3
	4	1.4	1.4	1.4	1.3	1.3

Unemployment Volatility (std)

δ

		2	3	4	5	6
u*	**3**	0.91	0.92	0.92	0.93	0.93
	3.25	0.89	0.90	0.90	0.90	0.91
	3.5	0.86	0.86	0.86	0.86	0.85
	3.75	0.82	0.82	0.83	0.83	0.83
	4	0.80	0.81	0.81	0.82	0.81

Bandwidth/STD inflation

δ

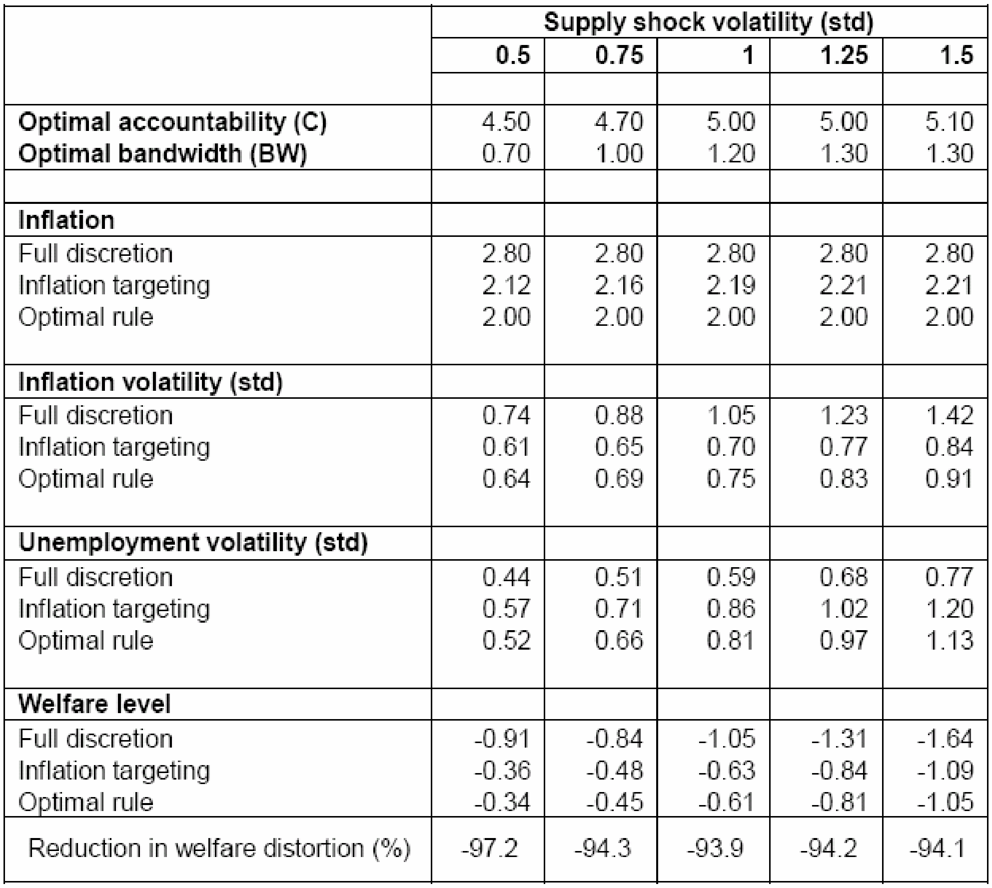
		2	3	4	5	6
u*	**3**	1.52	1.38	1.38	1.38	1.39
	3.25	1.77	1.64	1.65	1.65	1.51
	3.5	1.85	1.86	1.72	1.71	1.69
	3.75	1.89	1.89	1.78	1.78	1.77
	4	1.85	1.86	1.86	1.75	1.74

Welfare

δ

		2	3	4	5	6
u*	**3**	-0.66	-0.66	-0.67	-0.67	-0.67
	3.25	-0.65	-0.65	-0.65	-0.65	-0.66
	3.5	-0.63	-0.63	-0.63	-0.64	-0.64
	3.75	-0.62	-0.62	-0.62	-0.62	-0.62
	4	-0.61	-0.61	-0.61	-0.61	-0.61

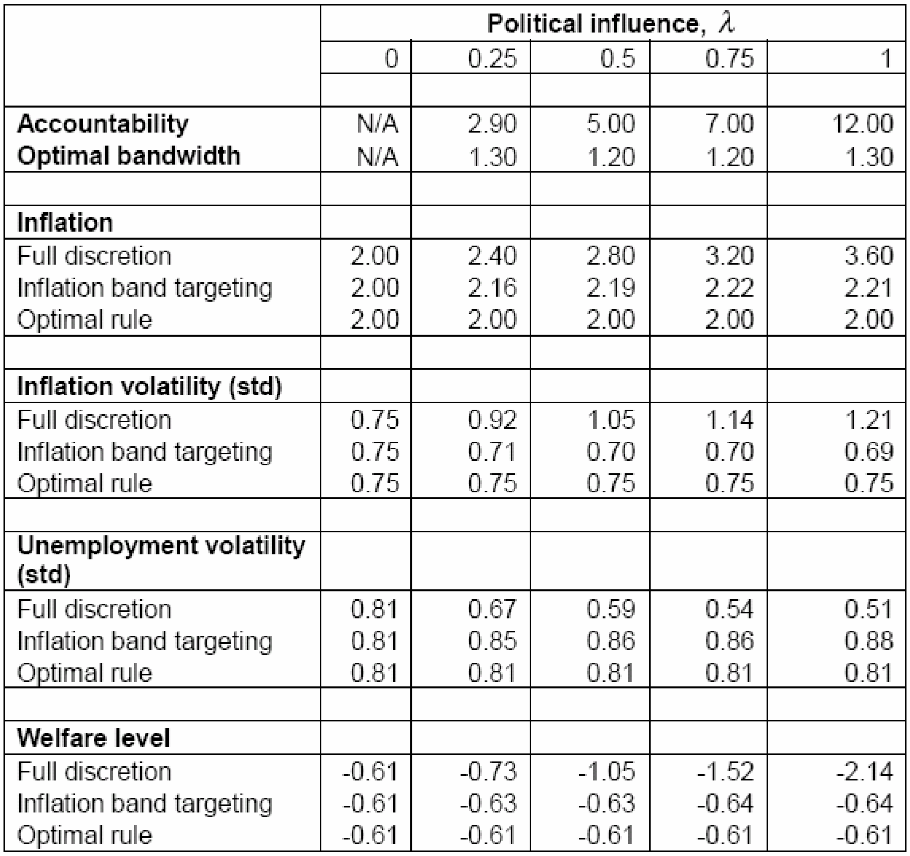
Table 2. Supply Shock Volatility and Optimal Design of Inflation Band Targeting



	Supply shock volatility (std)				
	0.5	**0.75**	**1**	**1.25**	**1.5**
Optimal accountability (C)	4.50	4.70	5.00	5.00	5.10
Optimal bandwidth (BW)	0.70	1.00	1.20	1.30	1.30
Inflation					
Full discretion	2.80	2.80	2.80	2.80	2.80
Inflation targeting	2.12	2.16	2.19	2.21	2.21
Optimal rule	2.00	2.00	2.00	2.00	2.00
Inflation volatility (std)					
Full discretion	0.74	0.88	1.05	1.23	1.42
Inflation targeting	0.61	0.65	0.70	0.77	0.84
Optimal rule	0.64	0.69	0.75	0.83	0.91
Unemployment volatility (std)					
Full discretion	0.44	0.51	0.59	0.68	0.77
Inflation targeting	0.57	0.71	0.86	1.02	1.20
Optimal rule	0.52	0.66	0.81	0.97	1.13
Welfare level					
Full discretion	-0.91	-0.84	-1.05	-1.31	-1.64
Inflation targeting	-0.36	-0.48	-0.63	-0.84	-1.09
Optimal rule	-0.34	-0.45	-0.61	-0.81	-1.05
Reduction in welfare distortion (%)	-97.2	-94.3	-93.9	-94.2	-94.1

The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\delta = 4$, and $\lambda = 0.5$

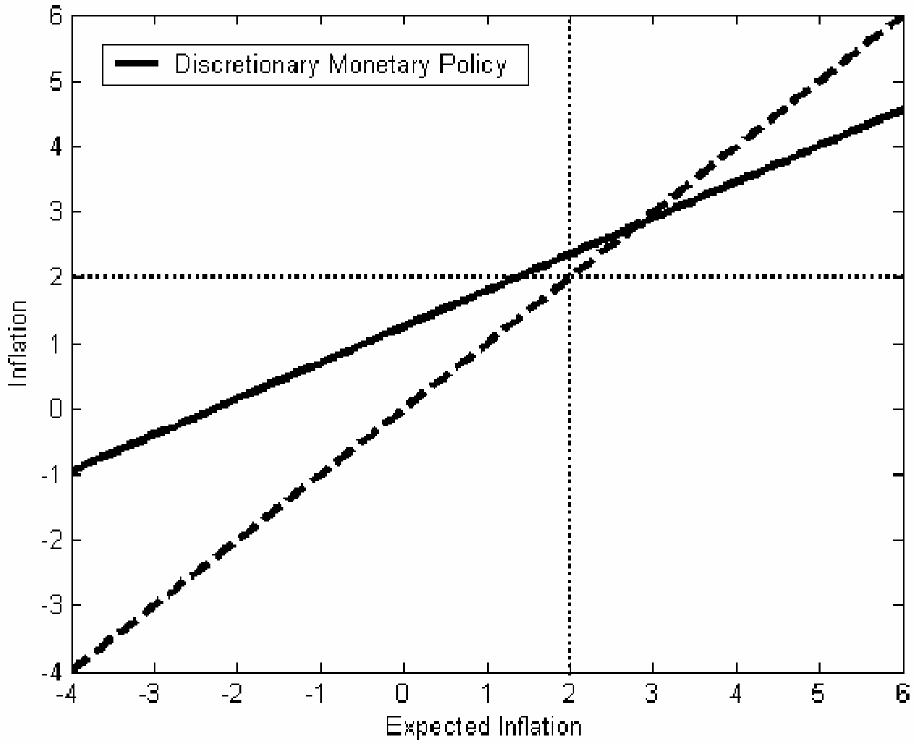
Table 3: Political Influence over Monetary Policy and Optimal Design of Inflation Band Targeting



	Political influence, λ				
	0	0.25	0.5	0.75	1
Accountability	N/A	2.90	5.00	7.00	12.00
Optimal bandwidth	N/A	1.30	1.20	1.20	1.30
Inflation					
Full discretion	2.00	2.40	2.80	3.20	3.60
Inflation band targeting	2.00	2.16	2.19	2.22	2.21
Optimal rule	2.00	2.00	2.00	2.00	2.00
Inflation volatility (std)					
Full discretion	0.75	0.92	1.05	1.14	1.21
Inflation band targeting	0.75	0.71	0.70	0.70	0.69
Optimal rule	0.75	0.75	0.75	0.75	0.75
Unemployment volatility (std)					
Full discretion	0.81	0.67	0.59	0.54	0.51
Inflation band targeting	0.81	0.85	0.86	0.86	0.88
Optimal rule	0.81	0.81	0.81	0.81	0.81
Welfare level					
Full discretion	-0.61	-0.73	-1.05	-1.52	-2.14
Inflation band targeting	-0.61	-0.63	-0.63	-0.64	-0.64
Optimal rule	-0.61	-0.61	-0.61	-0.61	-0.61

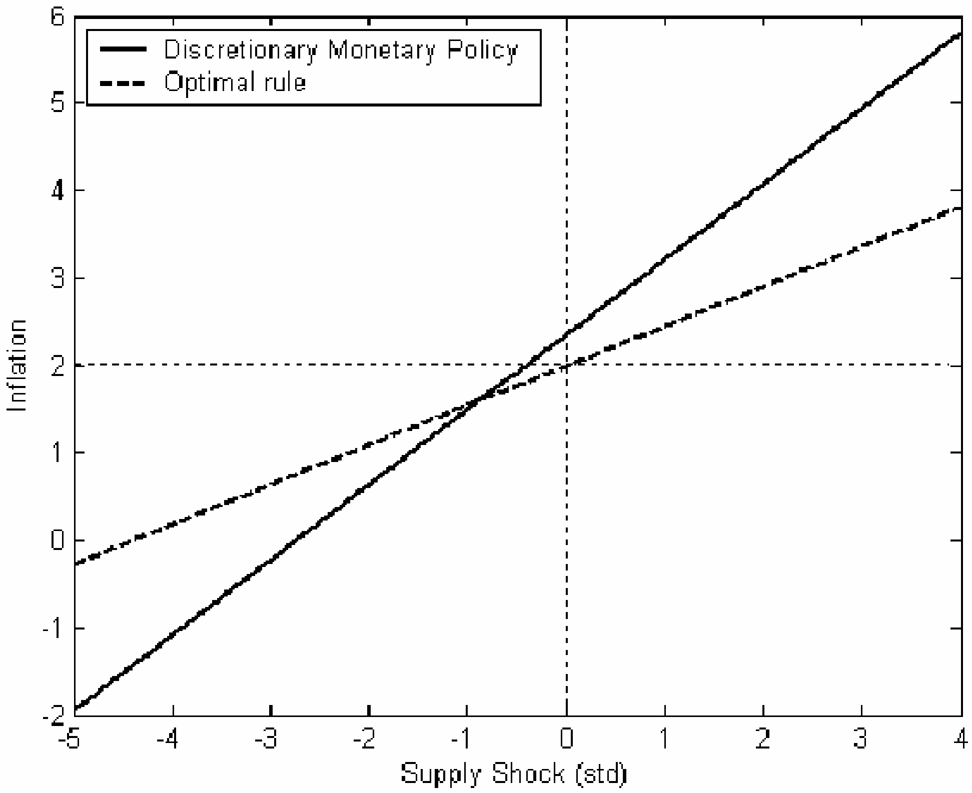
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, and $\delta = 4$

Figure 1. Equilibrium Inflation under Discretion



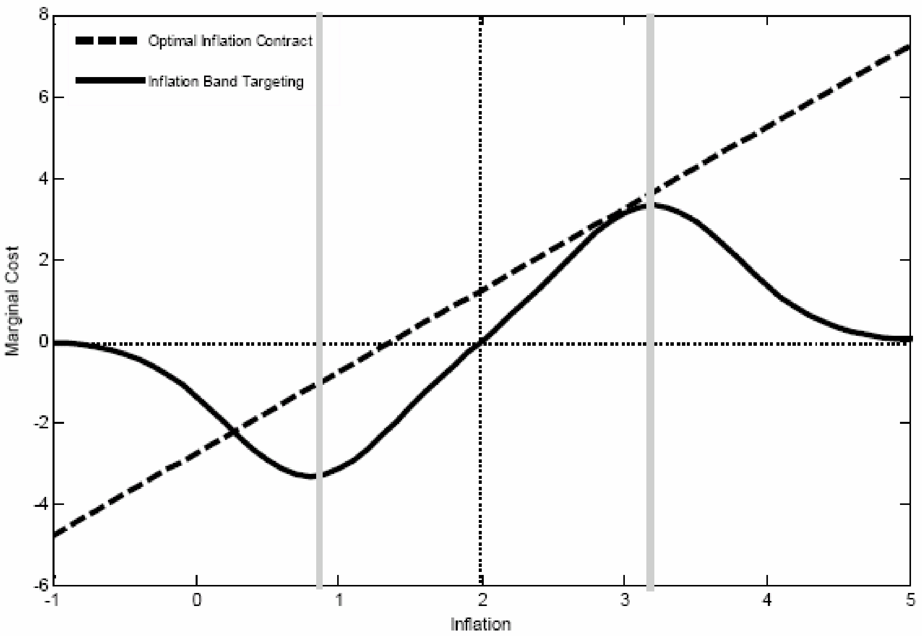
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, and $\lambda = 0.5$.

Figure 2: Shock Management under Pure Discretion and the Optimal Rule



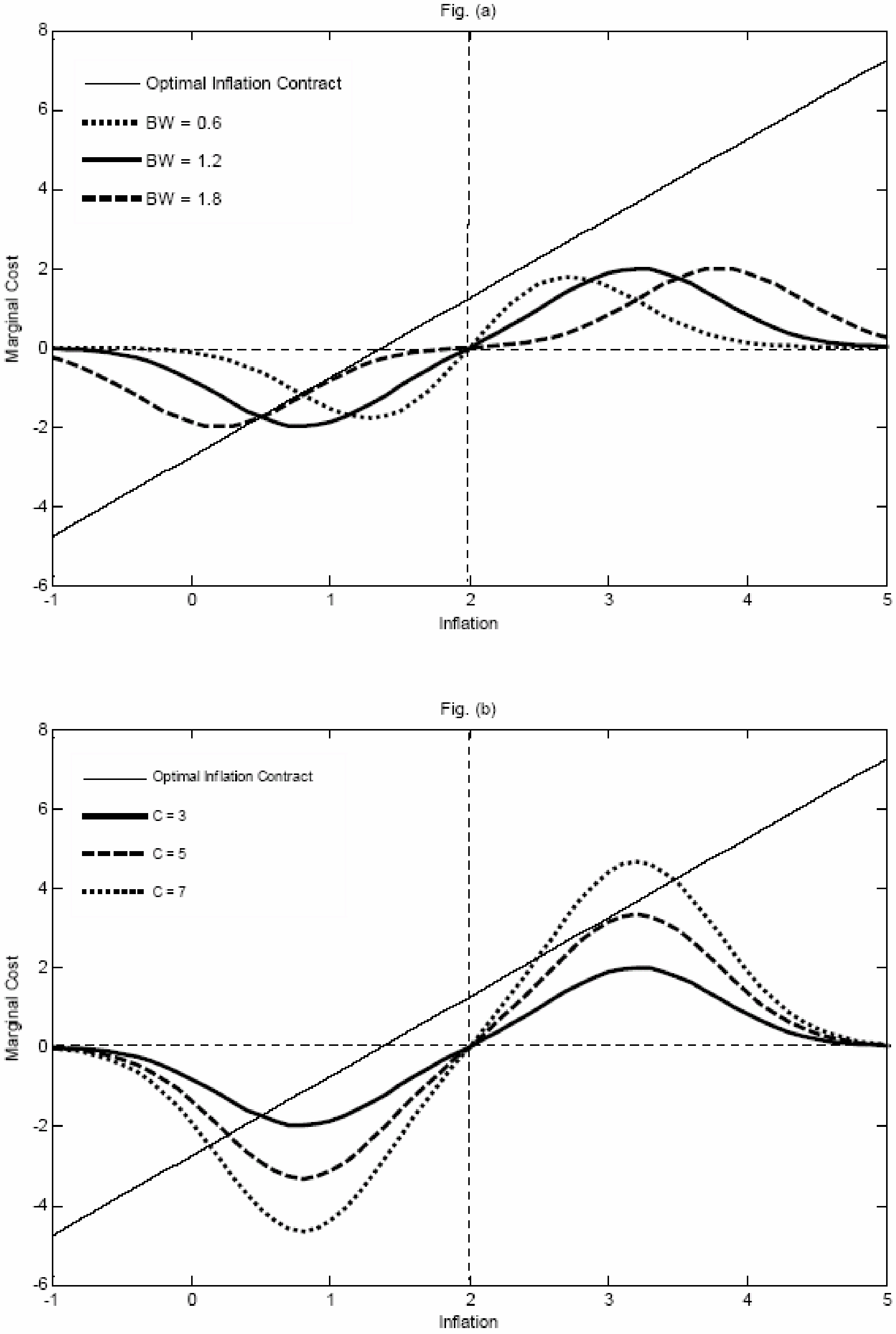
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, and $\lambda = 0.5$.

Figure 3. Marginal Cost Structure



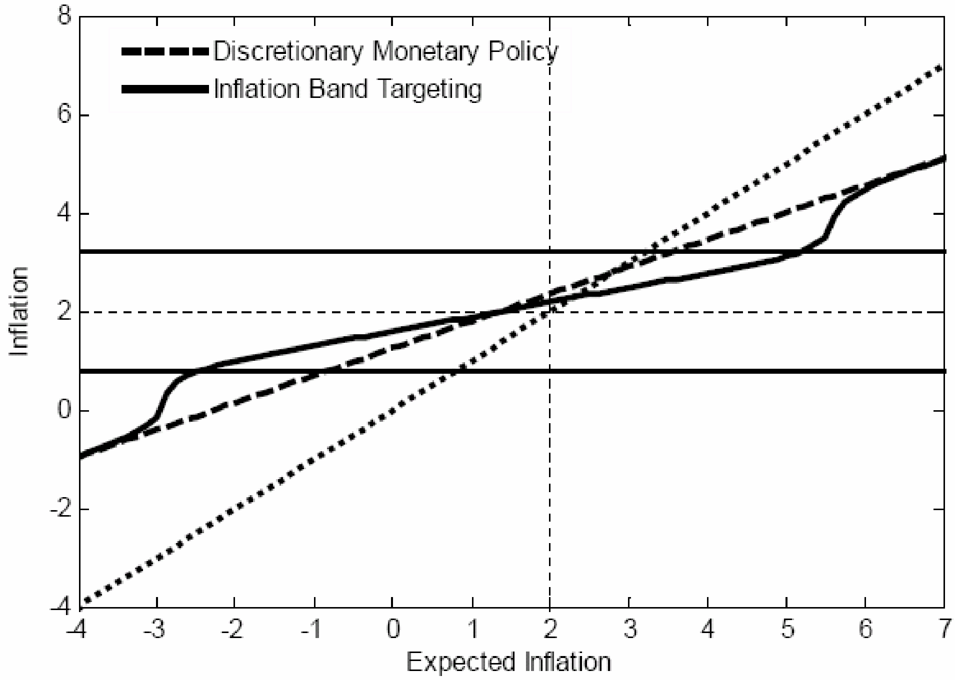
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$, $C = 3$, $BW = 1.2$

Figure 4. Marginal Cost Structure, Bandwidth and Accountability



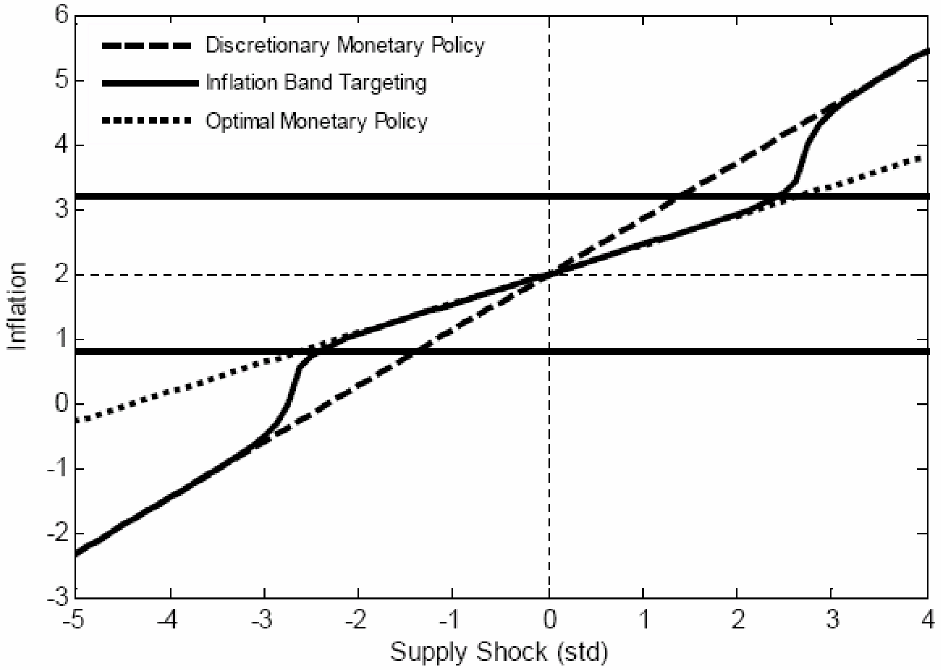
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$. For figure 4(a) the degree of accountability was fixed at $C = 3$. For figure 4(b) the bandwidth was fixed at $BW = 1.2$

Figure 5. Equilibrium Inflation under Inflation band Targeting



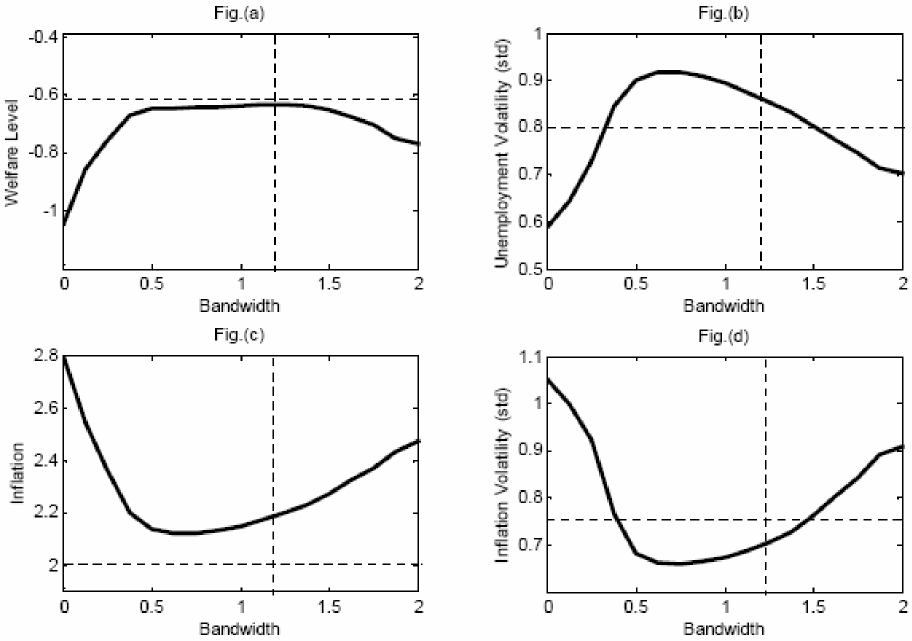
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$, $C = 3$, and $BW = 1.2$.

Figure 6. Shock Management under Inflation Band Targeting



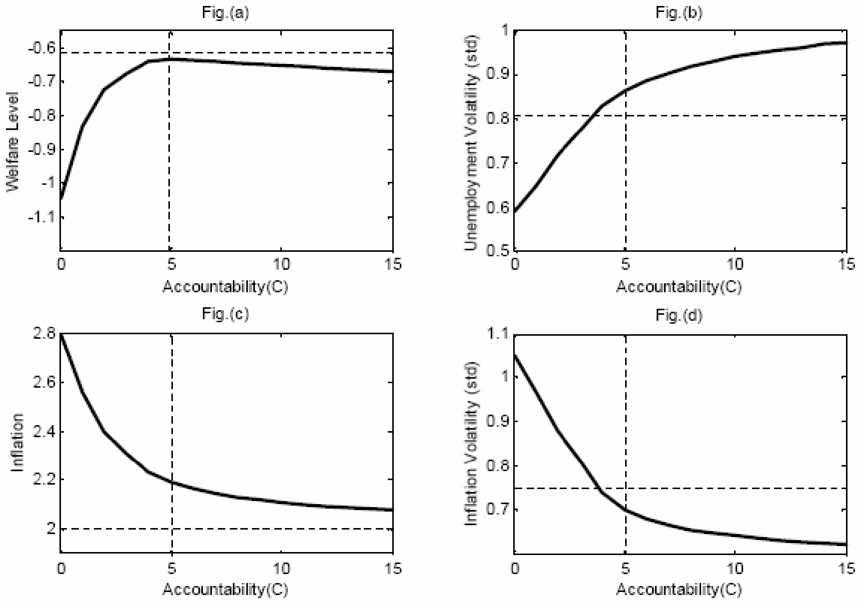
The following parameterization was used: $b = 0.64$, $u^* = 4$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$, $C = 3$, and $BW = 1.2$.

Figure 7. Optimal bandwidth given the Degree of Accountability



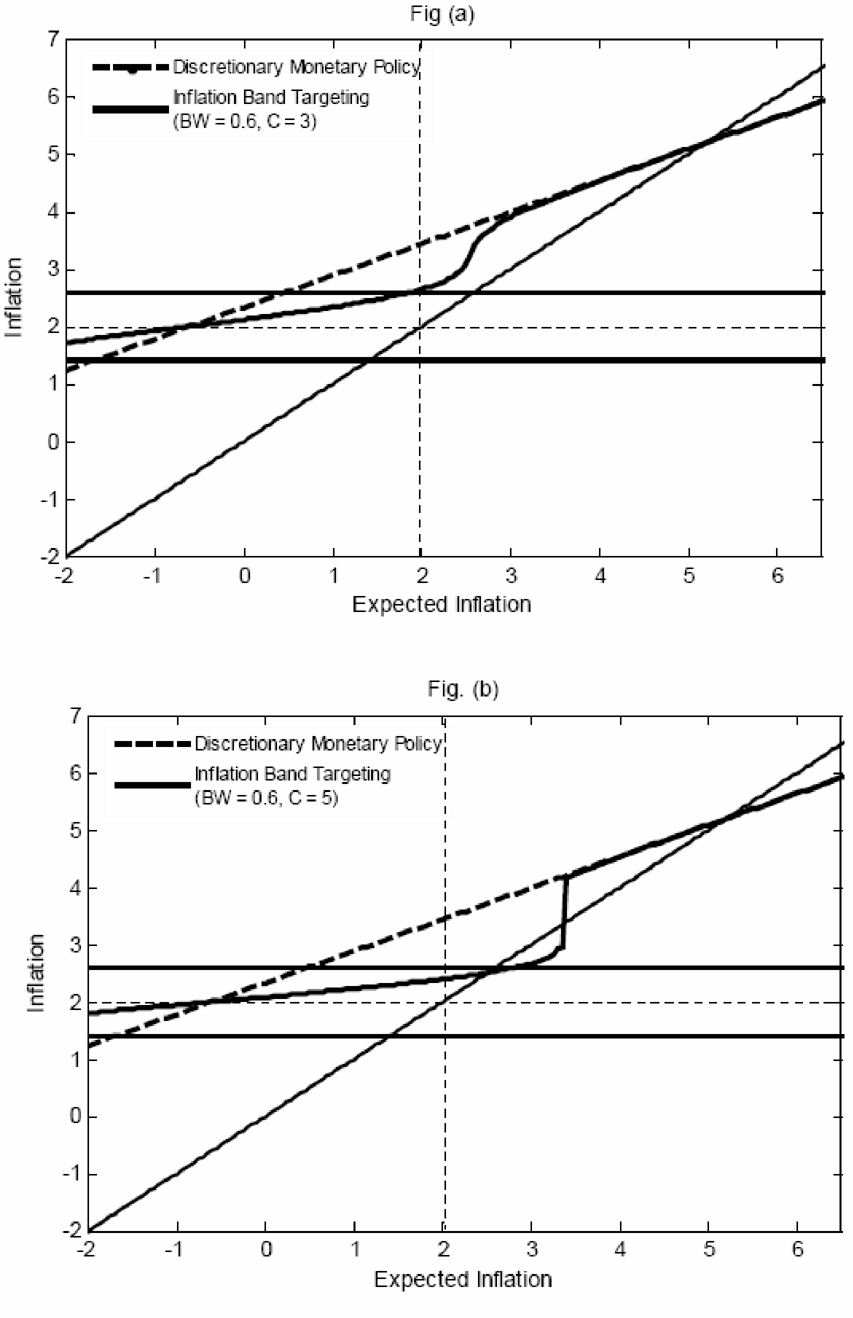
The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$, and $C = 5$.

Figure 8: Optimal degree of Accountability given the Bandwidth



The following parameterization was used: $b = 0.64$, $u^* = 3.5$, $u^n = 4$, $\omega_\pi = 1$, $\omega_u = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, $\lambda = 0.5$, and $BW = 1.2$.

Figure 9. Multiple equilibria under inflation band targeting



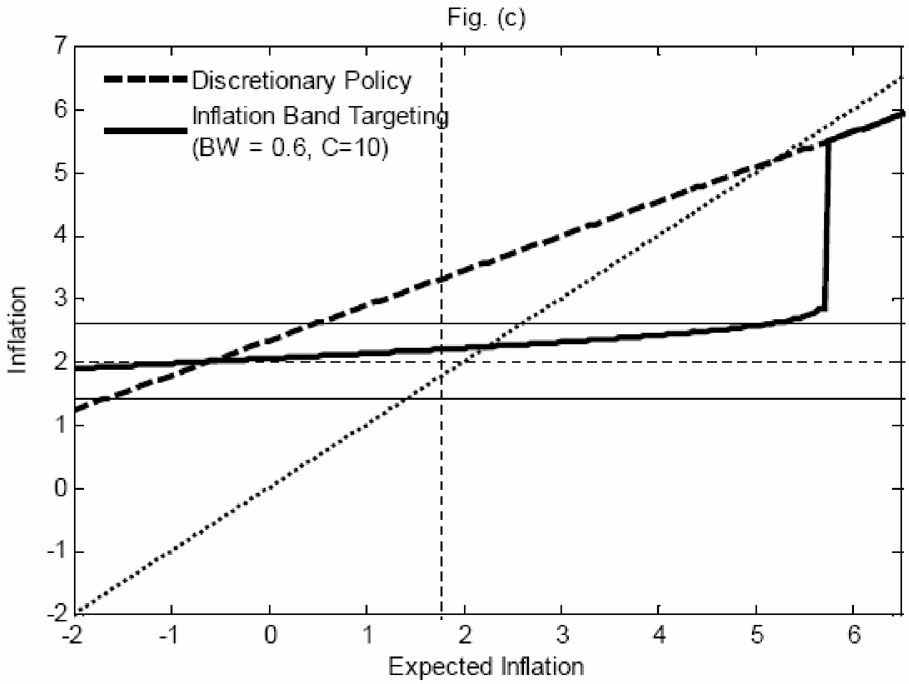


Fig. (c)
Discretionary Policy
Inflation Band Targeting
(BW = 0.6, C=10)
Inflation
Expected Inflation
7
6
5
4
3
2
1
0
-1
-2
-2
-1
0
1
2
3
4
5
6