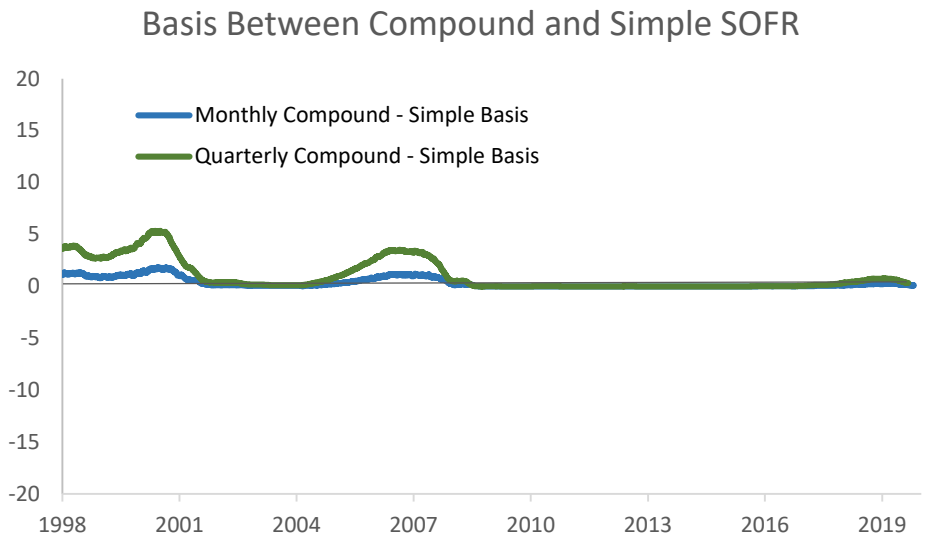
Appendix 1. Simple versus Compound Interest

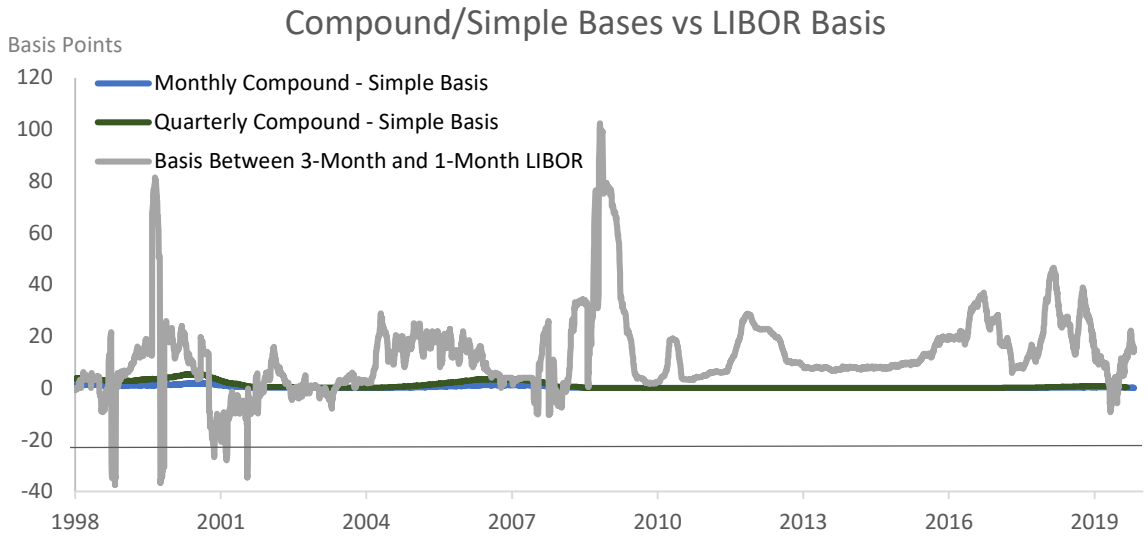
The ARRC conventions recognize that either simple or compound interest can be charged when using SOFR in arrears. As discussed in the User's Guide to SOFR, although compound interest will more accurately reflect the time value of money and will match the payment structure in derivatives and debt market, simple interest is in some ways operationally easier to implement, because daily interest accruals only depend on the principal outstanding at the time of accrual, while daily accruals under compound interest will additionally depend on the amount of unpaid interest (or, as discussed in Appendix 3, the cumulative compound rate of interest rates from the start of an interest period)

The ARRC expects that market participants will choose between simple or compounded interest, depending on the circumstances of each loan; however, many members of the ARRC's business loans working group expressed a preference for simple interest in arrears over compound interest in arrears for syndicated U.S. dollar business loans. Those who held this preference noted that the basis between simple and compound interest has historically been very small, and even in higher interest rate periods was a few basis points, as shown in the figure below:[1]



As shown in the next figure, compared to the basis between 1-month and 3-month LIBOR, which is relevant for loans that allow the borrower to move between different LIBOR tenors, the basis between simple and compound interest is essentially de minimis.

[1] As noted in the User's Guide to SOFR, the difference between compound and simple interest depends on the level of interest rates, because compounding interest charged on unpaid accrued interest will be smaller when interest rates are low, and it will depend on the length of the interest reset, because compound interest increases with the length of the interest period.

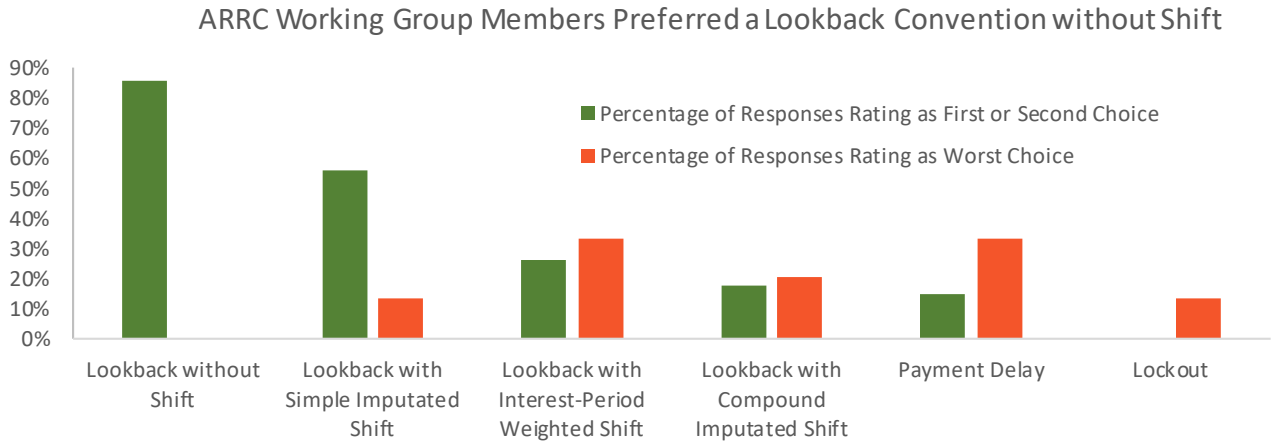


In addition to recognizing that the basis between simple and compound interest is fairly small, many working group members also recognized that loan and loan-trading systems are already able to handle simple in arrears loans, and believed that while vendors will offer systems to calculate compound interest (and some have already), that the need to update operational systems to allow compounding would take up time and resources that could be devoted to instead transitioning away from LIBOR more quickly.

Appendix 2. Lookbacks and Other Conventions for Timely Payment Notice

As discussed in the User's Guide to SOFR, there are a number of conventions to allow the borrower and lender sufficient notice of the final payment under an in arrears framework, including a *payment delay*, a *lookback*, or a *lockout*. Since the publication of the Guide, some alternative versions of the basic lookback structure were developed as the ARRC has considered potential conventions, including lookbacks *without observation shift*, *with observation shift*, *simple- or compound imputed shifts* or *interest-period weighted shifts*. We explain these conventions in more detail in this Appendix.[2]

As shown below, a lookback without observation shift was ranked as either the first or second choice by 86 percent of the working group members who expressed a view between the different conventions. A lookback with a simple imputed shift was the next most favored, but clearly less favored than the ARRC's recommendation, and other conventions garnered more negative ratings than positive.



With a payment delay, payment is due some number of days after the end of an interest period., and with a lockout, the final SOFR rate is fixed for the last few days of each interest period. Neither convention was viewed as well suited to the loan market. Although payment delays are the standard convention for derivatives and some working group members noted that there were a few examples of overnight bilateral loans that have used a payment delay in the past, these were not prevalent and there were no examples in the syndicated loan market. Working group members ultimately believed that a payment delay would require too many changes to the loan market. Likewise, although a lockout structure has been used in some FRN issuances, a lockout cannot be easily implemented in an environment where

[2] The ARRC's Syndicated Loan Conventions are for loans using an in arrears framework. Other loans based on SOFR may use an in advance framework, but the conventions for those loans are likely to be similar to the current conventions for LIBOR loans. An in advance payment structure references an average of the overnight rates observed *before* the current interest period began, while an in arrears structure references the rates during the current the interest period and is only know *at or near the end* of the period. Loans based on the 30-, 90-, or 180-day SOFR averages published by the Federal Reserve Bank of New York would generally tend to be set in advance, as would loans potentially based on a forward-looking term SOFR rate. Although loans set in advance offer operational advantages, a loan based on SOFR in arrears will reflect what actually happens to interest rates over the period and will therefore fully reflect movements in interest rates in a way that LIBOR or a SOFR-based forward-looking term rate will not.

early prepayment is possible, as it is in many loans, and would be difficult to integrate with trading amongst lenders.

For these reasons, the ARRC has preferred a lookback structure, and we focus on those conventions below. To explain what a lookback is, we first describe the payment structure of a loan without a lookback, and then describe the different versions of lookback structure that the ARRC considered.

In the terminology of these conventions, the *interest date* is the date that interest is applied for, while the *observation date* is the date that the SOFR rate is pulled from. Under compound interest, the daily SOFR rate is compounded across business days and the given SOFR rate applied over the number of calendar days until the next business day.[3] The distinction between business days and calendar days isn't as important with simple interest, but under compound interest it is a more important operational consideration.

Without a lookback, the interest date is equal to the observation date and interest is charged based on the SOFR rate that the Federal Reserve Bank of New York publishes for that business day. In the example below, interest on July 2 would be based on the SOFR rate for July 2 and would be applied for 1 day until Wednesday, July 3; interest on July 3 would be based on the SOFR rate for July 3 and because since July 4 is a holiday, Wednesday's rate would be applied for 2 business days until Friday, July 5.

FRBNY SOFR DATA		
DATE	RATE (PERCENT)	Calendar Days Until Next Business Day
Mon, Jun 24, 2019	2.39	1
Tue, Jun 25, 2019	2.41	1
Wed, Jun 26, 2019	2.43	1
Thu, Jun 27, 2019	2.42	1
Fri, Jun 28, 2019	2.5	3
Mon, Jul 1, 2019	2.42	1
Tue, Jul 2, 2019	2.51	1
Wed, Jul 3, 2019	2.56	2
Fri, Jul 5, 2019	2.59	3
Mon, Jul 8, 2019	2.48	1
Tue, Jul 9, 2019	2.45	1

No Lookback: The date that the SOFR rate is pulled from (the observation date) is the same date that interest is applied (the interest date) and applies until the next business day following the interest date.

Example: The rate for July 2 is applied on July 2 for one day, while the rate on July 3 is applied on July 3 for two days.

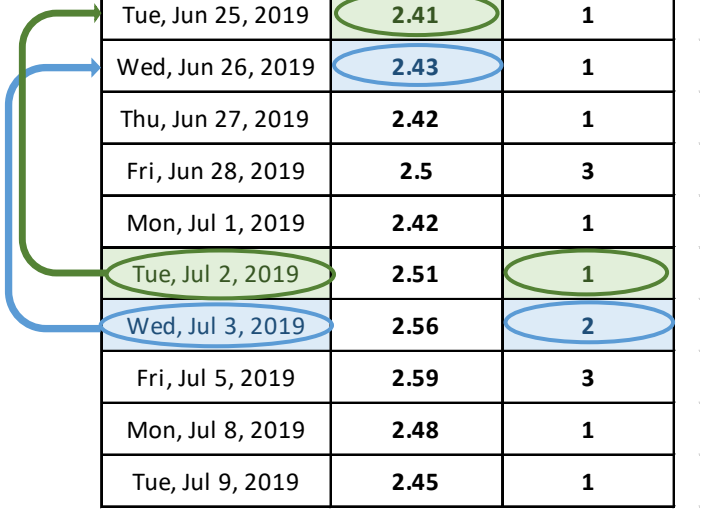
[3] SOFR is published on government securities trading days, as established by the Securities Industry and Financial Markets Association (SIFMA).

Because the SOFR rate for any given day is published on the following business day (the day that payment would be due on an overnight repo transaction or would be due on an overnight loan), without a lookback or some other convention to give more time for payment, the borrower would have at most a few hours to make final payment. We look at each of the different potential lookback structures in turn, but some readers may wish to only focus on the recommended convention of a lookback without observation shift.

Lookback without observation shift

A lookback gives counterparties more notice by applying the SOFR rate from some fixed number of business days prior to the given interest date. If the lookback is for k days, then the observation date is k business days prior to the interest date. In a lookback without an observation shift, all other elements of the calculation are kept the same and the reference to a previous SOFR rate is the *only* change made.

Continuing the example, using a 5-day lookback without observation shift in calculating interest for Tuesday, July 2, the SOFR rate for June 25 (5 business days prior to July 2) would be applied for 1 business day until Wednesday July 3, while in calculating interest for Wednesday, July 3, the SOFR rate for June 26 (5 business days prior to July 3) would be applied for 2 business days until Friday, July 5.



FRBNY SOFR DATA		
DATE	RATE (PERCENT)	Calendar Days Until Next Business Day
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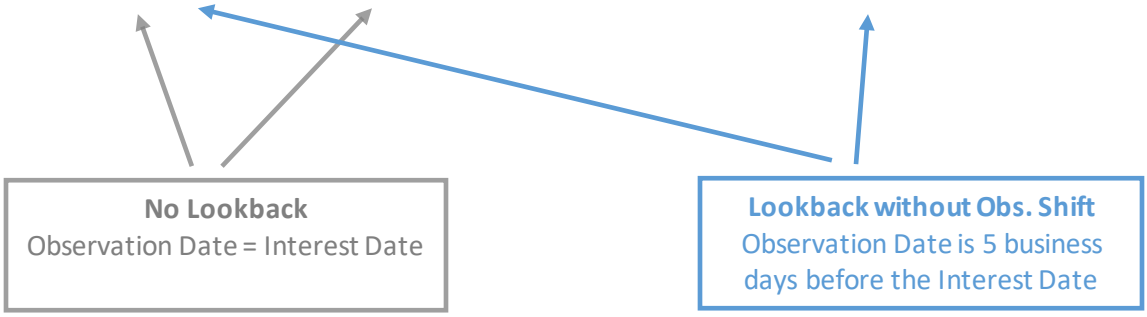
Wed, Jun 26, 2019	2.43	1
Thu, Jun 27, 2019	2.42	1
Fri, Jun 28, 2019	2.5	3
Mon, Jul 1, 2019	2.42	1
Tue, Jul 2, 2019	2.51	1
Wed, Jul 3, 2019	2.56	2
Fri, Jul 5, 2019	2.59	3
Mon, Jul 8, 2019	2.48	1
Tue, Jul 9, 2019	2.45	1

Lookback: The date that the SOFR rate is pulled from (the observation date) is k business days before the date that interest is applied (the interest date) and applies until the next business day following the interest date.

Example of a 5-business day lookback: The rate for June 25 is applied on July 2 for one day, while the rate on June 26 is applied on July 3 for two days.

The ARRC has released a set of spreadsheets along with these technical Appendices in order to aid market participants as they test their implmentation of various conventions. An example of a 5-business day lookback is included in the file ARRC BWLG Example - Lookback without Observation Shift.xlsx, and a segment of the spreadsheet is shown below. As in the example above, in order to implement a lookback without observation shift, the only change in calculations in the spreadsheet relative to no lookback is that the observation date is 5-business days earlier than the interest date.

		No Lookback			5-Business Day Lookback		
Interest Date (t)	# days rate applies (n_t)	SOFR Observation Date (t)	Relevant SOFR Print (r_t)	SOFR Effective Rate $i_t = \frac{r_t \cdot n_t}{N}$	Observation Date (t-5)	Relevant SOFR Print (r_{t-5})	SOFR Effective Rate $i_t = \frac{r_{t-5} \cdot n_t}{N}$
Mon, July 1, 2019	1	July 1, 2019	2.42%	0.00672%	June 24, 2019	2.39%	0.00664%
Tue, July 2, 2019	1	July 2, 2019	2.51%	0.00697%	June 25, 2019	2.41%	0.00669%
Wed, July 3, 2019	2	July 3, 2019	2.56%	0.01422%	June 26, 2019	2.43%	0.01350%
Thu, July 4, 2019							
Fri, July 5, 2019	3	July 5, 2019	2.59%	0.02158%	June 27, 2019	2.42%	0.02017%
Mon, July 8, 2019	1	July 8, 2019	2.48%	0.00689%	June 28, 2019	2.50%	0.00694%
Tue, July 9, 2019	1	July 9, 2019	2.45%	0.00681%	July 1, 2019	2.42%	0.00672%

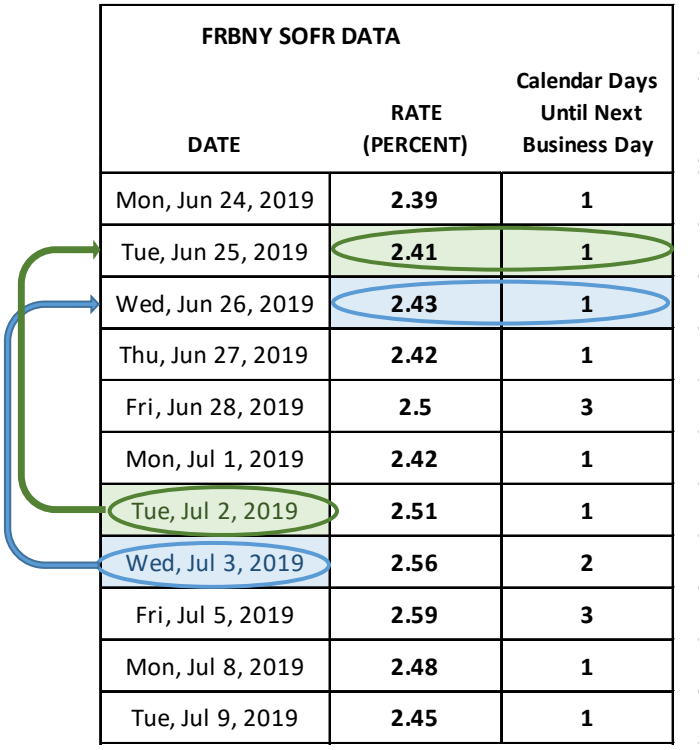


If the interest date is t, then a 5-day lookback will use the SOFR rate from the observation date t-5 (r_{t-5}) and it will apply that rate for the number of calendar days until the next business day following date t (n_t). The effective rate (i_t), which is the rate that is used in calculating daily accruals, is the SOFR rate on the observation date (r_{t-5}) multiplied by the number of days the rate applies for (n_t) and divided by the standard U.S. money market daycount convention of N =360.

Lookback with observation shift

A lookback *with* observation shift also applies the SOFR rate from some fixed number of business days prior to the given interest date, but in contrast to a lookback without a shift, it applies that rate for the number of calendar days until next business date following the *observation date*.

Continuing the example, using a 5-day lookback without observation shift in calculating interest for Tuesday, July 2, the SOFR rate for June 25 (5 business days prior to July 2) would be applied for 1 business day until Wednesday July 3, while in calculating interest for Wednesday, July 3, the SOFR rate for June 26 (5 business days prior to July 3) would be applied for 1 business day.



FRBNY SOFR DATA		
DATE	RATE (PERCENT)	Calendar Days Until Next Business Day
Mon, Jun 24, 2019	2.39	1
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Thu, Jun 27, 2019	2.42	1
Fri, Jun 28, 2019	2.5	3
Mon, Jul 1, 2019	2.42	1
Tue, Jul 2, 2019	2.51	1
Wed, Jul 3, 2019	2.56	2
Fri, Jul 5, 2019	2.59	3
Mon, Jul 8, 2019	2.48	1
Tue, Jul 9, 2019	2.45	1

Lookback with observation shift: The date that the SOFR rate is pulled from (the observation date) is k business days before the date that interest is applied (the interest date) and is applied for the number of calendar days until the next business day following the observation date.

Example of a 5-business day lookback with observation shift: The rate for June 25 is applied on July 2 for one day, and the rate on June 26 is applied on July 3 for one day.

A lookback with observation shift is one of the conventions that has been recommended by the ARRC for floating rate notes (FRNs).[4] However, syndicated loans have several complicating features that FRNs do not – principal can typically be repaid at any time, and syndicated loans are frequently traded between lenders and they do not trade clean.

The fact that principal may be repaid or that a lender may trade out of a loan before the end of an interest period makes implementing an observation shift more difficult in the loan market. For instance, in the example above, on July 3 interest is only charged for one day even though it would be two days until interest was paid. A lender who bought in to the loan on July 3 and sold out on July 5 may consider

[4] This convention is described under Two-Day Backward Shifted Observation Period and No Lockouts in the ARRC's SOFR Floating Rate Notes Conventions Matrix. See https://www.newyorkfed.org/medialibrary/Microsites/arrc/files/2019/ARRC_SOFR_FRN_Conventions_Matrix.pdf.

that they have been less than fully compensated given that they have provided some amount of principal for two days but only receive interest for one day. Or consider what was meant to be a monthly loan that began on July 8 but was repaid the next day. Under a 5-business day lookback with observation shift, the borrower would be charged for three day's interest based on the SOFR rate for Friday June 28, even though they had only borrowed money for one day and should therefore only be charged for one day's interest.

Without trading or without early repayment, these discrepancies would average out and would be inconsequential. Because principal is constant in FRNs (and because they trade clean, meaning that the purchaser receives the full coupon), an observation shift is more easily implementable. With trading and the possibility of early repayment, these kinds of discrepancies may be more problematic, and the ARRC Business Loans Working Group members felt that a lookback with observation shift would not be the most appropriate convention for the syndicated loan market.[5]

Other Potential Lookback Conventions

The Business Loans Working Group did discuss several variants of the observation shift that could avoid some of these problems, although they ultimately did not recommend them for syndicated loans: an interest-period weighted shift, a simple-imputed shift, and a compound-imputed shift. We briefly outline each:

Interest-Period Weighted Shift

As discussed above, the effective SOFR rate is used to calculate daily accruals. Without a lookback, the effective rate is

$$i_t = \frac{r_t \times n_t}{N}$$

and with a k-day lookback but no observation shift, the effective rate is

$$i_t = \frac{r_{t-k} \times n_t}{N}$$

As discussed further in Appendix 3, with no lookback or a lookback without observation shift, the unannualized cumulative compound rate of interest is

$$UCR_t = \left[\prod_{b=1}^{t}(1 + i_t) - 1\right]$$

[5] An analogy would be the difference between renting an apartment and staying at a hotel. Under a rental agreement, rent is the same each month even though some months have 28 days and others have 31 days, but the differences average out and people feel free to ignore them. In contrast, someone staying at a hotel is much more likely to take offense if they are charged for 3 days but only stayed 1 day or if they are charged a weekend rate when they stayed on a weekday.

and the equations and analysis in Appendix 3 can be used as they are, substituting whichever form of effective rate is appropriate. In both those cases, compounding is over the days in the interest period (which is defined in Appendix 3 as d_c, the sum of the n_t)

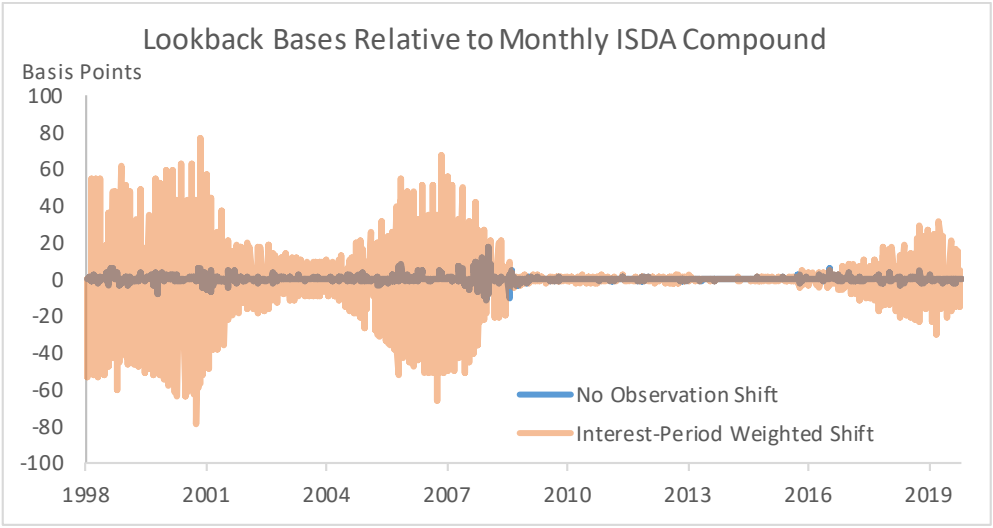
But using a lookback with an observation shift, the effective rate is

$$i_t = \frac{r_{t-k} \times n_{t-k}}{N}$$

and compounding is over the number of days in the observation period (which we will call d_{oc}). As noted, this can lead to a discrepancy between the number of days that interest is charged for and the number of days that the loan is outstanding or held. To correct this, an interest-period weighted shift re-weights the compounded averages to reflect the number of days in the interest period:

$$\left[\prod_{b=1}^{t}(1 + i_t) - 1\right] \cdot \frac{d_{ct}}{do_{ct}}$$

This equation is implicitly used to calculate coupons in some compound SOFR FRNs with lookbacks and observation shifts that specify an interest rate based ISDA's annualized compound SOFR formula and then apply the annualized rate to the number of calendar days in the interest period. The is adjusted lookback does accrue interest for the correct number of days in the interest period; however, compared to a lookback without observation shift, it can have substantial short-term basis relative to a standard SOFR OIS swap, as shown below.[6]



[6] The size of the basis may be surprising, but it reflects the fact that OIS swaps do not have a lookback and that over any given monthly period, the number of days in the observation period can differ from the number of days in the interest period by 3-4 days. For example, one might be 28 days and the other 31 days, which is roughly a 10 percent difference. When rates are high, a 10 percent difference can translate in to 50 basis points or more.

For FRNs, these choices matter less, because principal is constant and so any differences between whether, for example, one interest period has 89 days and another has 91, will tend to average out quickly. In a loan that can be repaid on held for only a short period of time, the calculations may not average out, although a borrower (or lenders) may not place much importance on hedging a loan that they could quickly repay or sell.

Another consideration that led the ARRC not to recommend an interest-period weighted shift is that it can be difficult to implement a daily floor under this convention. Daily accruals may in some circumstances be negative even if SOFR rates are positive or floored. The spreadsheet ARRC BWLG Examples - Other Lookback Options.xlsx demonstrates how to calculate an interest-period weighted shift, and also provides an example of a negative daily accrual under this convention.

Simple-Imputed Shift

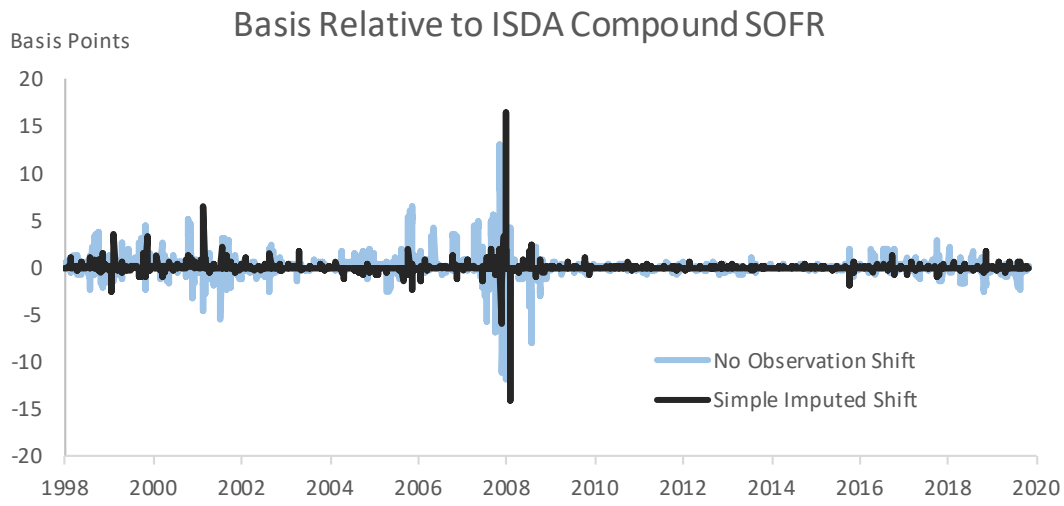
The problems with using an observation shift in the syndicated loan market arise when the number of calendar days between two observation dates are different than the number of calendar days between the corresponding interest dates. With a 5-day lookback, this would only occur around holidays. One way around this problem is to impute (or "fill-in") rates for those holiday dates. The most straightforward way to do this is to apply the rate observed for the date immediately preceding the holiday.

Shift with Simple Imputation (holidays imputed)

Interest Date (t)	Observation Date (t-5)	Relevant SOFR Print (r_{t-5})	# days rate applies (n_{t-5})	SOFR Effective Rate	SOFR Cumulative Compounded Effective Rate
Mon, July 1, 2019	Mon, June 24, 2019	2.39%	1	0.00664%	0.00664%
Tue, July 2, 2019	Tue, June 25, 2019	2.41%	1	0.00669%	0.01333%
Wed, July 3, 2019	Wed, June 26, 2019	2.43%	1	0.00675%	0.02008%
Thu, July 4, 2019	Thu, June 27, 2019	2.42%	1	0.00672%	0.02681%
Fri, July 5, 2019	Fri, June 28, 2019	2.50%	3	0.02083%	0.04765%
Mon, July 8, 2019	Mon, July 1, 2019	2.42%	1	0.00672%	0.05437%
Tue, July 9, 2019	Tue, July 2, 2019	2.51%	1	0.00697%	0.06135%
Wed, July 10, 2019	Wed, July 3, 2019	2.56%	1	0.00711%	0.06846%
Thu, July 11, 2019	Wed, July 3, 2019	2.56%	1	0.00711%	0.07558%
Fri, July 12, 2019	Fri, July 5, 2019	2.59%	3	0.02158%	0.09718%

The spreadsheet ARRC BWLG Examples - Other Lookback Options.xlsx also demonstrates how to calculate an a simple imputed shift. In the screen shot of the spreadsheet shown above, this convention would require calculating interest for July 4 (the calculation itself could take place on July 5, but interest would be compounded separately for July 3 and July 4) using a 5-day lookback to June 27, and in calculating interest for July 11, it would impute a rate for July 4 by using the July 3 rate. With these two rates filled in, the number of days in the observation period for a 5-day lookback would equal the number of days in the interest period.

While this convention does have somewhat less basis relative to a standard SOFR OIS swap than a lookback without observation shift, the differences are very slight – typically less than a basis point. At the same time, implementing this would require nontrivial changes to vendor and lender systems, and the modest improvement in basis did not seem sufficient to warrant such changes.



Compound-Imputed Calendar Shift

This would be essentially the same as method a simple-imputed shift, but rather than taking the last day's rate (which is akin to a simple interest concept) this convention would impute an implied daily compound rate based on the rate from the previous business day. To do this, if the rate on the previous business day before a holiday was r and there were n calendar days until the next business day, then the imputed daily compounded rate would be

$$\bar{r} = (1 + n \times r)^{\frac{1}{n}} - 1$$

This convention has slightly less basis than a lookback without observation shift relative to a standard OIS swap, but as with the simple-imputed shift, the reduction in basis is slight and adopting the convention would require nontrivial changes to vendor and lender systems. Additionally, BWLG members believed that it would be difficult to explain how the imputed rate had been calculated.

Appendix 3. Calculating Compound Interest - Compound Balance versus Compound Rate

ISDA's Compound SOFR formula is based on the following annualized rate calculation:

(1) $Compound\ Annualized\ Interest = \left[\prod_{b=1}^{T}\left(1+\frac{r_b \times n_b}{N}\right)-1\right]\frac{N}{d_c}$

Where

T = the number of *business days* in the interest period

d_c = the number of *calendar days* in the interest period[7]

r_b = the interest rate applicable on business day *b*

n_b = the number of calendar days for which rate r_b applies (on most days, n_b will be 1, but on a Friday it will generally be 3, and it will also be larger than 1 on the business day before a holiday). This can also be stated as the number of calendar days from and including business day *b* to but excluding the following business day.

N = the market convention for quoting the number of days in the year (in the United States, the convention for money markets is N = 360, while in the UK it is N=365).

And *b* represents a series of ordinal numbers representing each business day in the period.

In this section, we lay out the conditions under which this type of "compound rate" equation can be used, and how to calculate compound interest more generally. To do so, we define a few additional terms for a given business date *t*:

i_t = the effective interest rate for date *t*

P_t = outstanding principal for date *t*

A_t = the accumulated unpaid accrued interest for date *t before* any interest paydown

A'_t = the accumulated unpaid accrued interest for date *t after* any interest paydown

PD_t = the amount of any interest paydown (a negative number, so that $A'_t = A_t + PD_t$)

The equations below would work with either an effective interest rate based on a lookback without observation shift ($i_t = \frac{r_t \cdot n_{t-k}}{N}$) or with no lookback ($i_t = \frac{r_t \cdot n_t}{N}$), as would be the case with the payment delay used in derivatives.

[7] As discussed in more detail in section 2, the formula as written assumes that $d_c = \sum n_b$, which *will* be the case with the standard uses of the ISDA compound SOFR formula in derivatives, where notional principal is typically constant over an interest period and payment is made with a delay.

General Case: Compound Balance

Whereas under simple interest daily accrued interest depends only on the outstanding principal for that day:

$$A_{t+1} = A'_t + i_t \cdot P_t$$

with compound interest, daily interest accrual is charged both on outstanding principal and on accumulated unpaid interest:

$$(2) \qquad A_{t+1} = A'_t + i_t \cdot [P_t + A'_t]$$

This formula is the basic definition of compound interest – interest is charged both on outstanding principal and accumulated unpaid accrued interest. Within the ARRC Business Loans Working Group, this equation (2) has been termed the "Compound Balance" approach to calculating compound interest. The Compound Balance approach can be applied regardless of whether principal changes or whether some portion of interest is repaid during an interest period.

The daily interest accrual under the Compound Balance approach is simply calculated by applying the appropriate day's SOFR rate to outstanding principal and accrued unpaid interest:

$$A_{t+1} - A'_t = i_t \cdot [P_t + A'_t]$$

The ARRC has published a spreadsheet ARRC BWLG Compounding Methods Examples.xlsx containing examples of the different methods of calculating compound interest. A screenshot of the worksheet with an example of compound balance is shown below. Implementation requires keeping track of accumulated interest as shown in the screenshot:

	A	F	G	H	I	J	K
1	SOFR Rate and Principal/Paydown Information				Compound Interest Accrual Calculations		
3	Interest Date (t)	Principal (P_t)	Interest Paydown (PD_t)	Effective Rate $i_t = \frac{r_t \cdot n_t}{N}$	Accumulated Unpaid Interest *Before* Paydown (A_t) $A_{t+1} = A'_t + i_t \cdot [P_t + A'_t]$	Accumulated Unpaid Interest *After* Paydown (A'_t) $A'_t = A_t + PD_t$	Daily Base Interest Accrual $i_t \cdot [P_t + A'_t]$
12	July 9, 2019	$100,000,000.00		0.00681%	$56,400.74	$56,400.74	$6,892.30
13	July 10, 2019	$100,000,000.00		0.00683%	$63,210.14	$63,210.14	$6,809.39
14	July 11, 2019	$100,000,000.00		0.00669%	$70,047.79	$70,047.79	$6,837.65
15	July 12, 2019	$100,000,000.00		0.01967%	$76,746.92	$76,746.92	$6,699.13
16	July 15, 2019	$90,000,000.00	($9,642.87)	0.00683%	$96,428.68	$86,785.81	$19,681.76
17	July 16, 2019	$90,000,000.00		0.00686%	$92,941.74	$92,941.74	$6,155.93
18	July 17, 2019	$90,000,000.00		0.00686%	$99,123.12	$99,123.12	$6,181.38

Accumulated Unpaid Interest Before Paydown

Cell I18 = J17+H17*(F17+J17)

Accumulated Unpaid Interest After Paydown

Cell J18 = I18 + G18

Daily Accrual

Cell K18 = H17*(F17+J17)

Special Case: Compound Rate

While the Compound Balance approach can be applied generally, as discussed further in Box A.1, the "Compound Rate" approach should only be employed under specific conditions:

a) Principal remains constant within an interest period, or

b) If some portion of principal is repaid, then a corresponding proportion or accrued interest is repaid at the same time.

Under the specific conditions, the general formula can be simplified to the (non-annualized) version of ISDA's formula for Compound SOFR

$$(3) \qquad A_{t+1} = \mathrm{UCR}_t \cdot P_t$$

Where the term $\mathrm{UCR}_t = [\prod_{b=1}^{t}(1 + i_b) - 1]$ is called the Unannualized Cumulative Compound Rate.

Daily accrual can be calculated directly using this equation and equation for A'_t

$$A_{t+1} - A'_t = \mathrm{A}_{t+1} - \mathrm{A}_t - PD_t$$

but market participants have tended to prefer a variant of this calculation, the "Noncumulative Compound Rate" approach, which recognizes that the required relationship between that amount of interest paid down and any reduction in principal implies that this calculation for daily accrued interest can be simplified to:

$$A_{t+1} - A'_t = (UCR_t - UCR_{t-1}) \cdot P_t$$

	A	F	G	H	I	J
1	SOFR Rate and Principal/Paydown Information				Compound Interest Accrual Calculations	
3					Unannualized Cumulative Compound Rate (UCR_t)	Noncumulative Compound Rate Daily Base Interest Accrual
4	Interest Date (t)	Principal (P_t)	Interest Paydown (PD_t)	Effective Rate $i_t = \frac{r_t \cdot n_t}{N}$	$\mathrm{UCR}_t = \left[\prod_{b=1}^{t}(1 + i_b) - 1\right]$	$(UCR_t - UCR_{t-1}) \cdot P_t$
12	July 9, 2019	$100,000,000.00		0.00681%	0.06321%	$6,892.30
13	July 10, 2019	$100,000,000.00		0.00683%	0.07005%	$6,809.39
14	July 11, 2019	$100,000,000.00		0.00669%	0.07675%	$6,837.65
15	July 12, 2019	$100,000,000.00		0.01967%	0.09643%	$6,699.13
16	July 15, 2019	$90,000,000.00	($9,642.87)	0.00683%	0.10327%	$19,681.76
17	July 16, 2019	$90,000,000.00		0.00686%	0.11014%	$6,155.93
18	July 17, 2019	90,000,000.00		0.00686%	0.11701%	6181.38

Unannualized Cumulative Compound Rate

Cell I17 = (1+H17)*(1+I16) - 1

Daily Accrual

Cell J18 = (I17 – I16)* F17

The Compound Rate, and the Noncumulative Compound Rate equations are special cases of the Compound Balance approach. While the Compound Balance approach will correctly accrue interest under general conditions, if the special conditions are not met, that is if principal is repaid but interest is not, then the Compound Rate and Noncumulative Compound Rate approaches will not calculate accrued interest correctly.[8]

Although the Compound Balance approach is more general, many market participants have gravitated toward the Noncumulative Compound Rate approach to calculating interest accrual in part because the form of the equation is similar to the calculation under simple interest, in that a single rate (in this case the difference between today's and yesterday's cumulated compound rates) is applied to current principal.

Some loan systems may report an Annualized Cumulative Compound Rate

$$\text{ACR}_t = UCR_t \cdot \frac{N}{d_c}$$

or a Noncumulative Compound Rate,

$$NCR_t = (UCR_t - UCR_{t-1}) \cdot \frac{N}{n_t}$$

NCR will not correspond to any day's SOFR rate; even if SOFR is constant, the NCR will rise tend to rise over the interest period because it embeds both the SOFR rates paid on daily principal and the interest charged on accumulated unpaid interest.

[8] There are, however, two possible workarounds to fix this problem:

(i) treat any unpaid interest as a new loan starting at date *t*, while treating the original loan as if interest had been repaid on it and using the Compound Rate approach with it.

(ii) carry a separate set of internal calculations accruing interest on the amount of principal that would be outstanding if the borrower's payment was proportionately allocated between principal and interest reduction.

In either case, the workarounds effectively reproduce the Compound Balance calculations and simply using the Compound Balance approach directly may arguably be more straightforward.

Box A1. Conditions Where the Compound Rate Approach Correctly Accrues Interest

The ISDA Compound SOFR formula was created for a standard OIS swap, which have constant principal over an interest period., but the formula can be applied to a loan or other instrument for which principal is constant within an interest period.

The equation isn't too difficult to derive when principal is constant. First note that in the absence of any interest paydown, $A'_t = A_t$ and that if principal is constant ($P_{t+1} = P_t = P$) then the compound balance equation (1) can be written as:

$$(A_{t+1} + P) = (1 + i_t) \cdot (A_t + P)$$

Using this, and the fact that at the start of an interest period there is no unpaid accrued interest ($A_1 = 0$), one can recursively solve for accrued interest:

$$\begin{aligned}(A_2 + P) &= (1 + i_1)P \\ (A_3 + P) &= [(1 + i_2)(1 + i_1)]P \\ (A_4 + P) &= [(1 + i_3)(1 + i_2)(1 + i_1)]P \\ &\vdots \\ (A_{t+1} + P) &= \left[\prod_{b=1}^{t}(1 + i_b)\right]P\end{aligned}$$

Which gives equation (3), $A_{t+1} = [\prod_{b=1}^{t}(1 + i_b) - 1]P$

If the borrower repays some portion of principal within an interest period, then the compound rate methods will not generally calculate accrued interest correctly. However, there is a special case, when a proportionate share of accrued interest is repaid along with principal, in which the compound rate approach will still calculate the correct amount of accrued interest. To see that, assume that there has been no principal paydown or interest repayment up to time *t*, so that the compound rate formula holds up to that time. If the share of principal paid down is α, then:

$$P_t = (1 - \alpha) \cdot P_{t-1}$$

and for the compound rate equations to correctly accrue interest, it would require that the same share of accrued interest was also paid back at the same time so that:

$$A'_t = (1 - \alpha) \cdot A_t$$

With this proportionate interest paydown we have:

$$\begin{aligned}A'_t &= (1 - \alpha) \cdot \left[\prod_{b=1}^{t-1}(1 + i_b) - 1\right]P_{t-1} \\ &= \left[\prod_{b=1}^{t-1}(1 + i_b) - 1\right]P_t\end{aligned}$$

Using this with the compound balance equation

$$A_{t+1} = (1 + i_t) \cdot (A'_t + P_t) - P_t$$

gives

$$\begin{aligned}A_{t+1} &= (1 + i_t) \cdot \left[\prod_{b=1}^{t-1}(1 + i_b)\right]P_t - P_t \\ &= \left[\prod_{b=1}^{t}(1 + i_b) - 1\right]P_t\end{aligned}$$

Appendix 4. Floors

Although interest rate floors in derivatives and FRNs would generally be implemented as a floor on the compound average rate determined at the end of the period, the ARRC is recommending a daily floor for SOFR in arrears syndicated loans. Because these loans can be repaid early and because trading in syndicated loans is not clean, without a daily floor it is possible that the amount a borrower would pay in the instance of an early payment or the amount a lender would earn if they did not hold the loan for the entire interest period would be less than the intended floor.[9]

While this convention is different from some other products, it is straightforward to implement in a *new* SOFR in arrears loan. With either simple or compound interest in arrears, the daily SOFR rate would be floored at the desired rate level and then interest (either simple or compound) would be accrued on the floored rate while any accompanying margin would then be added as a simple interest accrual.

For *legacy* LIBOR loans falling back to SOFR, because LIBOR will convert to a spread-adjusted SOFR rate that takes in to account the historic differences between LIBOR and SOFR, the floor should be adjusted. If there is a floor for LIBOR, then the ARRC recommends that comparable floor for daily SOFR should be

$$(1) \quad SOFR\ Floor = Legacy\ LIBOR\ Floor - ARRC\ Spread\ Adjustment$$

Under simple interest, it is straightforward to see that this will ensure that sum of the floored Daily Simple SOFR rate and the ARRC's recommended spread adjustment will not fall below the original LIBOR floor, and it is equivalent to flooring the spread-adjusted Daily SOFR rate at the original LIBOR floor.

Although it is more involved to prove, this will also ensure that the combined annualized daily rate of accrual on Daily Compounded SOFR and the ARRC's recommended spread adjustment does not fall below the legacy LIBOR floor. Per the recommended treatment of the spread adjustment in a LIBOR loan that transitioned to Daily Compounded SOFR, the legacy loan falling back would compound the daily SOFR rate but would apply the spread adjustment as simple interest.[10] In order to implement this, the Daily Compounded SOFR rate would first be floored according to equation (1) and then compound interest would be calculated using this floored Daily Compounded SOFR rate, while the spread adjustment and any margin accompanying the loan would accrue as simple interest.

The ARRC has published an accompanying spreadsheet ARRC_BWLG Daily Floor Examples.xlsx that demonstrates how to calculate compound accrued interest on a legacy loan under these proposed conventions. The spreadsheet provides an example of a zero LIBOR floor and uses ESTR data rather

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[9] For a SOFR loan in advance, either based on the SOFR averages published by the Federal Reserve Bank of New York or a potential forward-looking term SOFR rate, the floor would not need to be daily because the same in advance rate would be used throughout the interest period.

[10] The ARRC's recommended spread adjustment to compound SOFR in arrears for business loans will be based on the five-year historical median of the difference between a given tenor of LIBOR and the corresponding compound average of SOFR. As such, the spread adjustment should not itself be compounded, because it already reflects the difference between LIBOR and a compound average of SOFR. Including the spread adjustment as part of the compound interest calculations of the loan would in essence compound that amount twice.

than SOFR, since ESTR has fallen below zero, and it shows how to implement the compounding calculations under either the Compound Balance approach or the Noncumulative Compound Rate approach (screenshot shown below).[11]

	B	D	E	F	H	I	J	K	L
1	SOFR Rate and Principal/Paydown Information					Compound Interest Accrual Calculations			
2	Observation Date (No Lookback)	ESTR Print (r_t)	Spread Adjustment for Legacy Loans	Legacy LIBOR Floor	# days rate applies (n_t)	Floored ESTR Rate	Effective Floored ESTR Rate	Unannualized Cumulative Compound Rate	Noncumulative Compound Rate Daily Base Interest Accrual
14	Sep 16, 2019	-0.450%	0.5%	0	1	-0.450%	-0.00125%	-0.01875%	-$375.78
15	Sep 17, 2019	-0.451%	0.5%	0	1	-0.451%	-0.00125%	-0.02000%	-$124.98
16	Sep 18, 2019	-0.552%	0.5%	0	1	-0.500%	-0.00139%	-0.02139%	-$125.25
17	Sep 19, 2019	-0.555%	0.5%	0	1	-0.500%	-0.00139%	-0.02278%	-$138.86
18	Sep 20, 2019	-0.553%	0.5%	0	3	-0.500%	-0.00417%	-0.02694%	-$138.86
19	Sep 23, 2019	-0.554%	0.5%	0	1	-0.500%	-0.00139%	-0.02833%	-$416.57

Floored ESTR Rate:

Cell I19: =
I19 = Max(D19,F19-E19)

[11] There are other potential methods to implement a legacy floor. One potential would be to increase the amount of the spread adjustment on any days that the SOFR rate fell below the rate in equation (1) rather than flooring the SOFR rate itself. Another would be a combination of the ARRC's recommendation and this alternative, in which the SOFR rate would be floored at the original LIBOR floor, and the spread adjustment would be decreased as necessary by the difference between SOFR and the LIBOR floor. Although either of these alternatives would be feasible and would provide an appropriate and consistent floor for a legacy LIBOR loan, the working group members believed that altering the static spread adjustment on a potentially daily basis could cause some confusion and would also require systems changes that would not otherwise be needed for any other purpose.